Journal
of
The Franklin Institute

EDITOR, HENRY BUTLER ALLEN, MET.E., D.Sc.
ASSISTANT EDITOR, JOHN FRAZER, PH.D.

Associate Editors:

WILDER D. BANCROFT, PH.D.
C. B. BAZZONI, PH.D.
ARTHUR L. DAY, SC.D.
R. EKSERGIAN, PH.D.
A. S. EVE, F.R.S.
PAUL D. FOOTE, PH.D.
W. J. HUMPHREYS, PH.D.
C. E. K. MEES, D.SC.
WILLIAM B. MELDRUM, PH.D.
HENRY C. SHERMAN, SC.D.
W. F. G. SWANN, D.SC.
HUGH S. TAYLOR, D.SC.
A. F. ZAHM, PH.D.
JOHN ZELENY, PH.D.

Committee on Publications:

G. H. CLAMER, CHAIRMAN
RICHARD HOWSON
C. L. JORDAN
LIONEL F. LEVY
MALCOLM LLOYD, JR.
HIRAM S. LUKENS
RICHARD H. OPPERMANN
WINTHROP R. WRIGHT

Vol. 245 **MARCH, 1948** **No. 3**

CONTENTS

A Study of Patent Policies in Educational Institutions, Giving Specific Attention to the Massachusetts Institute of Technology. Part I. ..... VINCENT LEE MCKUSICK 193

Construction of Alignment Nomogram from Empirical Data ..... LO-HO 227

Symmetrical Bi-Modal Frequency Curves ..... V. H. GOTTSCHALK 245

Notes from the National Bureau of Standards 253

The Franklin Institute Notes 257

Library Notes:

- Recent Additions 258
- Publications Received 259
- Book Reviews 260

Notes from the Biochemical Research Foundation 265

Manometric Determination of Phosphomonoesterase (Alkaline Phosphatase). CHARLES A. ZITTLE

Current Topics 226, 268

Published by
THE FRANKLIN INSTITUTE OF THE STATE OF PENNSYLVANIA
Prince and Lemon Streets, Lancaster, Penna., and
Benjamin Franklin Parkway at Twentieth St., Philadelphia 3, Penna.

DOMESTIC—EIGHT DOLLARS PER YEAR FOREIGN—NINE DOLLARS PER YEAR
(Foreign Postage Additional)
SINGLE CURRENT NUMBERS—ONE DOLLAR EACH
REPRINTS—FIFTY CENTS PER INDIVIDUAL INSTALLMENTS

Indexes to the semi-annual volumes of the JOURNAL are published with the June and December numbers

Paschall 'Specials'

It is easy to overlook many unusual features of Paschall Service that are not obvious to industrial users.

For example—Paschall Oxygen is used extensively by aviators for breathing in high altitudes because it is exceptionally dry (99.5% pure). To the industrial user this extra dryness is a safeguard against freezing on the job.

Again—in many hospitals, Paschall Oxygen is specified for patients in oxygen tents. Here, the professional demand for extra purity and clean cylinders is an assurance of more efficient operation in industry.

These are merely samples of the 'Specials' that create preference for Paschall Compressed Industrial Gases. Any Paschall Salesman can point out many more . . . Are you listening?

PASCHALL OXYGEN COMPANY

73rd Street & Woodland Ave., Philadelphia 42, Pa. • Phone: SAratoga 9-0405

★

OXYGEN • NITROGEN • HYDROGEN • ACETYLENE

S. S. Bertz & Co., 212 W. Lemon St., Lancaster • United Materials Co., 3rd & Gordon Sts., Allentown
Miller Welding Supply Co., 208 N. George St., York • J. W. Kennedy, 530 Perry St., Trenton
Ted Killinger, 228 N. 5th St., Reading • Morey Flux & Chemical Co., 1813 Williamson St., Wilmington

Journal
of
The Franklin Institute
Devoted to Science and the Mechanic Arts

Vol. 245 MARCH, 1948 No. 3

A STUDY OF PATENT POLICIES IN EDUCATIONAL INSTITUTIONS, GIVING SPECIFIC ATTENTION TO THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY.*

BY

VINCENT LEE McKUSICK,[1] S. M.

PART I.

ACKNOWLEDGMENT.

This writer appreciated greatly the encouragement given him by Dean John W. M. Bunker, by Mr. Melvin R. Jenney, by Mr. Nathaniel M. Sage, by Dr. Joseph W. Barker, and by Dr. Archie M. Palmer in this analysis of the university patent problem.

TABLE OF CONTENTS, PART I.

CHAPTER		PAGE
I.	The University Patent Problem	196
	The patent problem and its significance	196
	Statement of the problem	196
	Importance of the problem	197
	Organization and source material of this study	198
	The American patent system	199
	Fundamentals of the patent system	200
	The statutory law	200
	Patent application and Patent Office proceedings	201
	Reissues and disclaimers	201
	Infringements	201
	Conclusion	202
	Why should universities take patents?	202
	Common fears of patents on university discoveries	202
	Reasons for universities to hold patents	204
II.	The Patent Experience of American Colleges and Universities	206
	Scattered beginnings before 1930	207
	Patent policy developed for single spectacular inventions	207

* A paper presented in the graduate seminar in electrical engineering at Massachusetts Institute of Technology by the author.

[1] Student, Harvard Law School, Cambridge, Mass.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

Patent policy evolving from growth of institutional research 210
Concurrent experience in non-profit research organizations 211
Summary to 1930 212
Experience since 1930 213
Reason for a definitive patent policy 213
Motivation for formation of university research foundations 214
University patents and the recent interpretation of the anti-trust laws 214
Present patent practice in American universities 216
Policy toward patent exploitation as an income source 216
Patent ownership 218
Treatment of inventions resulting from outside-sponsored research 220
Patent-administering agencies 221
Reward to the inventor 222
Licensing policy 223

While engineers and scientists employed in industrial laboratories have always been conscious of the importance of patents, their colleagues in university laboratories have only recently become convinced of any need for patenting their discoveries. The amount of research performed in universities has increased many times in recent years, and the influence of this research upon American economic life has become so immediate that scientists and administrators have been forced to work out patent policies governing the use of inventions made under university auspices. The problem of forming such patent policy involves both the *external* relations of the university with industry, government, and the general public and the *internal* relations between the university and its staff members. Because only the university can give equitable consideration to the best interests of all the parties concerned, upon it alone falls the responsibility for proper management of campus inventions.

University scientific workers commonly fear the practice of patenting because of the possible effect of commercialism upon the academic atmosphere of the campus laboratory and upon the traditional non-profit, tax-free nature of the university. On the other hand, the university's chief purpose for holding patents is to encourage public use of its inventions; university patents can protect the manufacturer who takes the initial risks of developing and commercializing the invention. Patents also prevent misappropriation of university discoveries by others than the true inventors, and can earn income for support of continued research. In deciding whether to take patents, educational institutions must weigh such advantages and disadvantages. University scientists and administrators now generally accept patents as proper means for protecting laboratory discoveries and for encouraging their public use.

Before 1930 a few universities took patents and developed them for income. Single spectacular discoveries, such as Insulin at the University of Toronto and Vitamin D at the University of Wisconsin, were

patented to control the quality of these public health products and to earn profits for the university. After 1930 financial necessity, as well as tremendous growth in the absolute magnitude of the research performed in universities, turned academic interest to patents. By 1948 nearly all educational institutions have established policies toward patents, and several more are actively exploiting patents for profit. The University of Wisconsin has been notably successful financially, having collected seven million dollars from one patent in fifteen years; but it has fallen into practices which were ruled to be violations of antitrust laws. The patent experience gained by other universities in less spectacular ways indicates fairly definitive answers to several important questions of patent policy. Among these questions are: how shall the equities in staff inventions be divided between the university and the inventor? how shall inventions from outside-sponsored research be handled? what is a proper reward for the inventor? what form of licenses shall be given to manufacturers? and what organization for patent management can best serve the interests of the university and of the public?

The patent policy formulated by the Massachusetts Institute of Technology in 1932 reserves to the Institute rights to all inventions toward which the Institute has made substantial contributions in money or staff time. A Faculty Patent Committee determines the equities in inventions. An administrative Committee on Patent Management, using Research Corporation of New York as its agent, directs the commercialization of Institute patents. Research Corporation, a nonprofit organization originally formed to earn research funds by the Cottrell electrostatic-precipitation patents, has thirty-five years of experience in managing patents assigned to it by universities.

The chief internal problem of university patents is one of dividing the equities in inventions. The university has both a legal and an ethical right to inventions toward which it contributes, and it can manage patents more advantageously than can the individual faculty member. The inventor should receive consideration, however, through financial reward by a percentage of the royalties and through his participation in managing the patent on his invention.

In meeting the external problem, the university must balance considerations of public relations against the desire to earn income for support of scientific research. Even though exclusive licenses would frequently advance the general welfare, public opinion limits the university to giving only nonexclusive licenses. Thus, the only way available to the university for maximizing patent income is to increase efficiency in patent management. Research Corporation has demonstrated that it can induce manufacturers to take its patents on a nonexclusive basis and can administer patents in keeping with a university's

objectives, while "insulating" that university from direct legal and publicity difficulties.

SELECTED BIBLIOGRAPHY.

American Association for the Advancement of Science, "The Protection by Patents of Scientific Discoveries," supplement to *Science*, Vol. 79, no. 1, 1934, 40 pp.

Conference on Medical Patents, Transcript, *Journal of American Medical Association*, 113: 327–336, July 22, 1939, and 113: 410–427, July 29, 1939.

Engineering College Research Association, *Proceedings of the Annual Meeting*, June 22, 1944, Cincinnati, Ohio. "Symposium on Engineering Research Foundations or Corporations," pp. 13–82.

GRAY, GEORGE W., "Science and Profits," *Harpers*, 172: 539–549, April, 1936.

PALMER, ARCHIE M., "University Patent Policies," *Journal of the Patent Office Society*, 16: 96–131, Feb., 1934.

POTTER, ANDREW A., "Research and Invention in Engineering Colleges," *Science*, 91: 1–7, Feb. 5, 1940.

Technology Review, "Patenting of Institute Inventions," 39: 348–350, June, 1937.

U. S. Department of Justice, unpublished monograph, "Investigation of Patent Practices and Policies in Educational and Non-Profit Organizations." 29 pp. and 28 pp. Appendix.

CHAPTER I.

THE UNIVERSITY PATENT PROBLEM.

A. *The Patent Problem and its Significance*

1. *Statement of the Problem*

What shall universities do with the results of the scientific research conducted within their walls? What shall be the relation of the university to the campus research worker? These questions together with their many implications are now the concern of every educational institution embarked upon a research program in science and engineering.

In an economy such as ours in which invention and technological innovation are controlling factors, the end goals that universities set and the methods they use in managing the results of research have vast importance. Will university discoveries go forgotten and unused on laboratory shelves? Will they be left for monopolization and misuse by anyone? Or, will the university assume responsibility for bringing valuable inventions to public use? If the university uses patents to guide the public uses of its discoveries, what will be the consequences to our economy and to the traditional position of the university in our society?

Within the university, where scientific research has formerly been only incidental to the main function of teaching, research has now become a major function in its own right. Collegiate research which in the past has centered around individual efforts is becoming institutionalized; group research not dissimilar to that pursued in industry is growing on many campuses. In this new situation to what degree should the university control the research worker? What rights should the university have in valuable campus inventions? How will the research worker be rewarded?

The two general problems of university research might be differentiated as the external ones and the internal ones. The external problems involve the university in its relations to the public; the internal ones, its relations with its staff members.

More questions have been raised here than have yet been answered by university administrators, let alone analyzed thoroughly in a brief paper. However, a major part of the answer will be found when universities decide what to do about patents. It is with this specific phase of the more general questions proposed above that this study concerns itself.

Outside the walls of the university the American patent system has directed the use of inventions ever since there was a United States. Dr. Frank Jewett [1] has called the patent system "a powerful force in our growth from an insignificant agricultural and trading federation to the most powerful industrial nation on earth." Most educational institutions until about 1930 ignored the patent problem, but in recent years a fuller realization of the social and economic consequences of university research has forced them to come to grips with the problem. As Dr. Archie Palmer [2] has written, "the consequences of lack of [patent] policy are serious; they are not avoided by evasion [of the problem.]"

Wherever scientific research is conducted, the end product is likely to be a "new and useful art, machine, manufacture, or composition of matter," [3] which is by legal definition a patentable invention. Although the university scientist usually conceives of an inventor as a "tinkerer" or a "cut-and-try specialist," in reality and indeed before the law all scientific research workers are inventors of the highest order. While it is true that a newly-found law of nature cannot be patented,[4] (and to this extent pure research, in pushing back the frontiers of human knowledge, does not produce patentable inventions), nevertheless a process or apparatus for making use of a law of nature may well be patentable. Thus, universities that conduct scientific research are sources of patentable discoveries.

2. *Importance of the Problem*

The significance of the university patent problem is demonstrated by the scale of research activity going on in every sizable educational institution with a scientific or technical curriculum. The Massachusetts Institute of Technology is embarked upon a post-war research program of $6,000,000 annually. That is not at all atypical of the scale

[1] Ref. 4, p. xlv. The reference numbers in this and succeeding footnotes refer to items in the general bibliography, which will appear in the April issue of this JOURNAL.

[2] Ref. 40, p. 5.

[3] Ref. 43.

[4] Ref. 14, p. 111.

of operation of campus laboratories throughout the country. Even in 1940, before wartime research expenditures began, some $32,000,000 was spent annually by universities for research in the natural sciences.[5] These figures cannot indicate the research that is conducted as an adjunct to teaching.

Both industry and government have long realized that educational institutions have the staff, the equipment, and the environment particularly suited to profitable research. Since the War, industry, having more funds for research than ever before, and the government, realizing the importance of scientific research to the national defense, are pouring great sums of money into university laboratories. In past years philanthropists attracted by the non-profit nature of the university have contributed to the support of scientific research that will benefit the public in more general ways.

In view of the magnitude of campus research the implications of university patent policy in our economy are far-reaching. Universities, in one extreme, might become through control of technological advances the holding companies of American industry. Or, unbeknown to them, in their patent laxity they could be made the tools for the concentration and monopolization in industry. Or, as a third potential danger, in reaction from the rigors of the commercialism outside campus laboratories, universities might withdraw entirely from direct responsibility for contributing the technological life-blood to the American economic society. In the triangular area delimited by these three apexes of danger, university scientists and administrators are working out research policy.

B. *Organization and Source Material of This Study*

The object of this paper is the analysis of the patent policies and procedures at American universities and colleges with view to applying this experience to the specific problem at the Massachusetts Institute of Technology. As far as is warranted by the data available an answer will be attempted for the two questions of which the patent problem is the specific manifestation. How shall the university control the use of its discoveries and what is the proper relation between it and its research workers?

Chapter II will trace the development of university patent policies historically to indicate the motivation for their initiation and will compare and analyze separate university policies in regard to several logical

[5] Ref. 3, p. 80. The above-mentioned research funds of universities are lent significance by comparison with the $234,000,000 and $70,000,000 spent by industry and government, respectively, for technical research in 1940. However, governmental and industrial research was almost exclusively applied research. About $23,000,000 is estimated as spent in pure research by universities in 1940, as compared with only $16,500,000 total expenditures in that field by industry and government. (See ref. 3.)

subdivisions of the patent problem. With the experience of American universities generally as background, Chapter III will treat the patent developments at M.I.T., summarizing the Institute patent policy in subdivisions similar to those used for comparing other universities in the preceding chapter. Chapter IV will turn to procedural detail in explaining and analyzing how the Institute policy has been carried out; the management of specific patents will be studied to illustrate the M.I.T. policy and to permit more complete analysis. The final chapter will make an overall evaluation of university patent policy, drawing such conclusions as are possible with particular application to the Institute.

The data for this study have by necessity come largely from such diverse original sources as talks with individual administrators of university patent policies and numerous publications of individual universities. Only two general analyses of university patent policy have been published, one by Dr. Archie M. Palmer [6] in 1934 and another by Dr. Andrew A. Potter of Purdue University [7] in 1940. Both studies were limited in coverage and are now out of date. Realizing the importance of the patent problem with the magnified scale of postwar scientific research, the National Research Council has recently conducted a comprehensive survey of patent policy in educational institutions under the direction of the same Dr. Palmer mentioned above. Through the kindness of Dr. Palmer this writer has had access to the files of data collected for the National Research Council survey.

Before attacking the central consideration of the paper, two foundation stones must be laid. A resumé of the workings of the American patent system, particularly of Patent Office procedure, is necessary to an understanding of the problem faced by universities. Secondly, it is necessary to recognize frankly the traditional fear of patenting among scientific workers at universities, to analyze the basis of that fear, and to consider what compensating factors impel universities to protect their discoveries.

C. *The American Patent System* [8]

An intelligent analysis of the patent problems of universities is impossible without a rudimentary knowledge of the American patent system—the mechanics of its operation, its relation to research and inventions, and its economic and social results. To weigh its merits and faults is beyond the scope of this paper since a complete evaluation of the American patent system would involve complex economic, political, sociological, philosophical, and historical considerations. However,

[6] Ref. 40.

[7] Ref. 27.

[8] This abbreviated statement of the operation of the patent system is based on Berle and DeCamp's "Inventions and Their Management" (Ref. 2).

one can profitably strive to answer the question: What is this patent system which is part of the economic and political environment in which university research is conducted?

Although the patent system has been under heavy attack in recent years, nevertheless, its essential features will probably remain for many years to come. The National Patent Planning Commission [9] two years ago reported, "It is the Commission's conviction that this recognition of the exclusive ownership of intellectual property has fully realized the intent of the Constitution on behalf of the public and should be continued." The Commission recommended several changes to improve the workings of the patent system, while preserving its intrinsic worth. Scientists and engineers whether in universities or in industry will do well to understand fully the mechanics of patenting.

1. *Fundamentals of the Patent System*

The Constitution laid down the objective of the patent system in stating:

> The Congress shall have the power to promote the progress of science and the useful arts by securing for limited times to authors and inventors the exclusive rights to their respective writings and discoveries.

In return for revealing the details of his invention to the public, the inventor receives a grant of monopoly for a period of seventeen years.

The American patent system sets out to work for the public good, that is, to create more inventions and make better use of them, by three means;

(a) By encouraging inventors through the profit motive.
(b) By causing complete and immediate disclosure of the details of inventions, and
(c) By providing financial protection for manufacturers in undertaking expensive industrial developments of inventions.

The primary reason for the patent system is the advancement of the public good, not reward to the inventor.

2. *The Statutory Law*

The Federal law provides that:

> Any person who has invented or discovered any new and useful art, machine, manufacture, or composition of matter, or any new and useful improvements thereof, or who has invented or discovered or asexually reproduced any distinct and new variety of plant other than a tuber-propagated plant . . . may . . . obtain a patent therefor.

Principles of nature, however basic to technological advance, are not patentable; nor are mere ideas. As a court decision has put it, "Invention consists in the conception of a function, and the selection of means whereby the function can be operatively carried out."

[9] Ref. 26, p. 743.

Even though an inventor may devise something new and useful before anyone else in this country knew of or used the particular device in question and before it was described in any patent or printed publication anywhere in the world, he may be refused a patent because of two so-called statutory bars: either publication anywhere or public use or sale in this country one year or more before the filing date of the inventor's application for the patent.

The patent itself contains, besides a grant from the government, drawings illustrating at least one embodiment of the invention, a specification summarizing the state of the art and describing the invention in detail, and a set of claims, each of which describes a novel combination from the use of which others than the inventor are excluded.

3. *Patent Application and Patent Office Proceedings*

The patent application is prosecuted in the Patent Office as a negotiation between the examiner representing the public and the attorney representing the inventor. The application consists of one or more drawings, the specification, and the claims, which the inventor hopes to be included in the issued patent. For it to be valid, the patent must conceal nothing.

An *interference* proceeding may be necessary in the Patent Office in order to determine the priority of invention between two or more individuals claiming substantially the same patentable invention. The burden of proof is on the "junior party," the last alleged inventor to file application for patent. The junior party's chance of success is estimated to be one in seven or eight; thus, early application is important. In interference proceedings complete and witnessed records kept from the date of conception on are important in establishing priority of conception and diligence in reducing the invention to practice. The procedure in handling interferences is an extremely technical matter and is the chief cause for delay in the issuance of a patent.

4. *Reissues and Disclaimers*

An inventor has the right to surrender within one year an issued patent which through an oversight omits an important claim or which makes too broad claims, and he may then apply for a "reissue" patent. The reissue will have the same expiration date as the original patent. The inventor may also narrow the claims of his patent by a "disclaimer," which must be granted by the Patent Office.

5. *Infringements*

Someone has said that a patent gives the right to litigate. Indeed, a patent gives *only* the right to prevent an infringement, that is, to exclude all others from making, using, or selling the patented invention.

In deciding an infringement suit the Federal courts must consider the scope of the patent in question and its applicability to the article that allegedly is infringing it. Infringement suits may be brought either to recover damages, or to obtain an injunction against further infringement. Damages are now paid in few cases. Infringement suits are expensive—$10,000 is the average cost—and are warranted only in cases where considerable infringement has occurred or is likely to occur.

6. *Conclusion*

This brief explanation of patent procedure is intended to point up some of the factors involved in the mechanics of patenting. Carroll L. Wilson [10] has correctly stressed the ever present danger of costly patent litigation. States he:

> The problem of drawing careful specifications and claims is one requiring a large amount of time and thought as well as familiarity with patent law and practice. The issuance of a good patent which has promise of some importance is usually the result of extensive negotiations with the Patent Examiner. Procedure in handling a patent during interference is complicated and likely to be lengthy and expensive. Because of the large percentage of litigated patents which are found invalid, there is no strong presumption of validity in an unadjudicated patent. This means that in order for a patent to be generally recognized as valid, expensive litigation is frequently necessary.

The complication and expense involved in prosecuting patents is a serious liability that faces universities and the individual staff member. Assurance that a patent can earn net income for anyone is small.

D. *Why Should Universities Take Patents?*

Without begging the question before all the available facts are presented, at least a partial answer must be essayed at this point to the traditional opposition to the patenting of university discoveries. What are the common arguments against university patents? In contrast to these arguments, what basic factors in the American patent system make it advisable for universities to concern themselves with patents?

1. *Common Fears of Patents on University Discoveries*

Frequently research men in university laboratories think of patents as being unethical, or at least a bit off-color. Because of altruism or because of their interpretation of professional ethics, they object to the personal monopoly that is attendant on the practice of patenting; [11] this objection they then confuse with the decision not to patent at all. They think the publication of the details of their discoveries is sufficient.

Secondly, men interested in research often fear that secrecy, competition, and personal jealousies detrimental to the free-to-all atmosphere of their laboratories will be introduced with the practice of patent-

[10] Ref. 38, pp. 13–14.

[11] Dr. Bush's statement, ref. 39, p. 84.

ing.[12] Only two years ago Dr. Rabi,[13] Columbia University physicist and Nobel prize winner, testified before a Senate Committee that "a patent-minded colleague in our department would in time find that he has few scientific friends. We like to discuss matters freely and it gives us the jitters to feel that someone is going to rush off and patent some idea which comes up." Closely allied with this fear that patents will cause dissension in the community of scientists is a mistaken belief[14] that patents place a legal stricture on other scientists who subsequently do fundamental research in the same field.

A third common fear expressed by scientists[15] is that patenting university inventions will direct research away from fundamental research and into lines that promise quick and sure commercial reward. To quote Sir Walter Fletcher,[16]

> It will be difficult for them [research workers] not to feel that the university [which exploits patents] will be more inclined to reward by pay or promotion him who makes some addition to knowledge of an immediately profitable kind rather than him who works for the sake of knowledge itself.

This redirection of research aims might come about in either of two possible ways: by motivating scientists to enter developmental research in hope of obtaining patent profits or by causing university research funds to be allocated to projects with the best commercial possibilities. The first possibility is not very dangerous, for certainly the motivation of university scientists is too deep to be affected particularly by the illusory prospect of profits from patents. The second possible method by which patents might redirect university research aims is more dangerous, but can be consciously avoided in administration of research funds. Some scientists fear, furthermore, the commercialization attendant upon patenting of university discoveries will result in scientific slipshodness or outright dishonesty in evaluating competing products if such dubious practices will pay off.

Finally, both the professional status of the scientist and the position of the university in public opinion might be harmed, some believe,[17] by court litigation, competition with industrial laboratories, and other difficulties that might accompany patents. As one writer[18] has expressed it,

> One of the greatest glories [of scientific research] is its intellectual integrity and independence—but can this reputation continue unsullied in the clash of competitive sales campaigns of patented commodities, infringement suits, and other contentions of the marketplace in which the financial interest of the research institution is on one side of the dispute?

---

[12] Ref. 14, p. 11 and ref. 42, p. 4.
[13] Ref. 46, p. 976.
[14] Ref. 7, p. 332.
[15] Statement of Dean Langsdorf, Washington Univ., ref. 42, p. 3.
[16] Ref. 13, p. 548.
[17] Ref. 17, p. 21; ref. 24, p. 1317; ref. 40, p. 114.
[18] Ref. 18, p. 548.

The public might express very directly its disdain for the university which actively seeks patent royalties by letting it live on its earnings, by withholding contributions for research, and perhaps by revoking the tax exemption privileges of the universities. If events ever took this turn, educational institutions would become nothing more than business corporations and ultimately the value of a staff member would be judged by the commercial value of his discoveries.

These arguments have been expressed too frequently and too sincerely to be ignored. Only study of the patent experience of educational institutions will show if the difficulties envisioned have actually developed. However, there are factors intrinsic in the American patent system which answer some of the objections listed above and which make university patenting especially desirable for the public good.

2. *Reasons for Universities to Hold Patents*

Failure to patent, in the first place, will effectively hinder public use of many inventions. Contrary to the popular belief, an invention or scientific discovery is not given to the public simply by telling the public about it. Elihu Thomson, a famous inventor in the electrical industry and once Acting President of M.I.T., said cryptically, "Publish an invention freely, and it will almost surely die from lack of interest in its development. . . . Patent it, and if valuable, it will be taken up and developed into a business." [19] To put an invention on the market, and thus to make it of practical benefit to the public, involves in many cases sizable financial outlay in developmental work, an expense which a manufacturer cannot undertake if all others can outstrip him in competitive position by avoiding these initial expenditures.

The following case illustrates how "dedication to the public" has a result opposite to that intended: L. R. Cleveland, a research fellow at Johns Hopkins University, invented a method for sterilizing fruit juices by treatment with oxygen under high pressure.[20] The National Research Council, donor of the fellowship, considered it valuable enough to patent, but then dedicated it to "the free use of the people of the United States"; the sterilization method has never been used.[21] This instance is cited solely as one case where a university discovery went unused when offered freely to all comers. Admittedly, other economic and technological considerations may have been factors in its neglect; cause and effect is difficult to establish completely in social events of this type. Nevertheless, the major reason for existence of the patent system, reaffirmed after careful study by the distinguished National Patent Planning Commission [22] some two years ago, is its encouragement of public

[19] Ref. 38, p. 1505.
[20] Ref. 28, p. 19.
[21] Ref. 39, p. 25.
[22] Ref. 26, p. 743.

use of invention through protection for venture capital. This reason operates with equal force, whether the invention is made on a university campus or elsewhere.

Secondly, the inventor's sense of ethics should demand that he concern himself with the quality of the product derived from his discovery. "In the case of a new drug or process the possibilities of dilution or contamination or distortion are endless, and by means of a patent the inventor can insure that whatever is offered as the invention is technically right." [23] The need for quality control is illustrated by the experience of Dr. Babcock,[24] the University of Wisconsin professor who invented the Babcock test, still widely used for determining the butterfat content in milk. The test is good only to the degree that the glass test bottles used are accurately calibrated. Babcock, believing that the people of the State of Wisconsin "were entitled to all the benefits derived from its use," did not patent the discovery, and so had no control over manufacturers who threw machines with uncalibrated glassware on the market and almost sent the test into the discard. To cite an instance of a public health discovery, the control of the quality of Insulin by a committee of the University of Toronto, where Dr. Best and Dr. Banting made the discovery, was possible through the patent; proceeds from Insulin licenses supported control laboratories, as well as additional research to improve the product.[25] At the time of the discovery no laws sufficient to protect the public in regard to the quality of Insulin existed in either the United States or Great Britain.[26] Patent control effected by a public institution fills a gap in the protection of the public welfare. There has never been any question as to the wisdom of the Insulin control.

Furthermore, the educational research man should not provide fertile ground for the "patent pirate," who does his research in the current scientific publications, and who hastens to patent the discoveries of others. Unfortunately the same exactness and completeness of description is not employed in scientific publication as is demanded in a patent specification.[27] As the result, a shrewd patent thief can establish his claims of "novelty," obtain a patent, and restrict use of the invention to his own profit. Patents taken by the university discoverer preclude the possibility of monopoly misuse. The foregoing arguments were summed up by President Compton [28] in his annual report of 1932: Responsibility does not "end with mere publication of a patentable, scientific discovery or invention."

[23] Ref. 13, p. 543.

[24] Statement of Dr. H. L. Russell, Director of Wisconsin Alumni Research Foundation, ref. 39, p. 29.

[25] Ref. 4, pp. 85–86.

[26] Ref. 39, pp. 99–100.

[27] Ref. 28, p. 1319 ff.; ref. 24, pp. 22–23.

[28] Ref. 21, p. 101.

As a further argument for university patenting, the large university is in a better position to develop and protect the patent right than most individual staff inventors. The legal mechanisms which wealthy, established concerns may use to delay, hamper, and sometimes invalidate the patent rights of individual inventors are well known. As long as such inequality before the law remains a fact of our society, the individual inventor is fortunate to share the strength of an institution, which will administer his patent in accordance with aims similar to his own and will return to him a portion of the income.

Further advantages derived from patenting university discoveries will be investigated later in this study and are mentioned here only for completeness. By patenting its discoveries the university responsible for the original research protects itself from being stymied in further development and research in the same technical field. Also, income accruing from patent licenses may support further university research and education; and outstanding scientists that are often lost to higher-paid industrial positions are encouraged by participation in royalty earnings to remain in university work.[29] It has long been recognized that royalties from book sales protected by the copyright law have gone far in encouraging university professors to be productive in book-writing; unlike books which are easily commercialized by publishing houses, patents are put to commercial use only with difficulty.[30] In this patent commercialization the university can be of help to the individual staff inventor. A final incidental benefit of university patents comes from the discussion and dissemination of new ideas that accompany the complete disclosures made in patent specifications. In other words, patenting may constitute an important part of the university's publication responsibility.

CHAPTER II.

THE PATENT EXPERIENCE OF AMERICAN COLLEGES AND UNIVERSITIES.

At least two reasons make it advisable to study the development of patent policies in American universities generally before turning to a study of the specific development at the Massachusetts Institute of Technology. The experience of comparable institutions in their patent dealings will form a basis for critical analysis of the M.I.T. patent program. Through the years the men responsible for the patent policies at the Institute constantly exchanged experiences with other universities. Thus, the development of the M.I.T. patent policies was an outgrowth of the general university experience.

---

[29] Dr. A. R. Olpin, formerly Director of the Ohio State University Research Foundation, makes a strong case for university patents in conjunction with cooperative research with industry. Ref. 24, pp. 22–23.

[30] Ref. 42, p. 2.

A. *Scattered Beginnings Before 1930*

In the beginning university patent policy was one strictly of "hands-off." Traditionally the university believed that the invention was the private property, and the patent the private concern, of the university staff member. All the forces of inertia, all the common fears [1] of patenting held by the academicians, caused the colleges to ignore patents or else to shrug them off as a personal problem of the inventor. As legal precedent for this stand the university could point to the policy, still almost universally accepted, that professors should be allowed to copyright their own books and to realize the profits from such copyrights.[2] Galileo, and not the University of Padua, with which he was associated, applied to the Doge of Venice in the 16th century for a monopoly on a device for lifting water.[3] To return from this slight excursion into Renaissance history—this same situation existed in all American universities at the beginning of the twentieth century, in spite of the fact that the colleges and universities of the last century spawned many patentable ideas such as the electro-magnet of Henry at Princeton, the Babcock butterfat test at the University of Wisconsin, and the telegraph-line loading coil invented by Prof. Michael Pupin at Columbia. The first two inventions were not patented; the third made a millionaire of its inventor.

Universities were called upon to formulate a positive policy toward patents in the first instance by spectacular circumstances. Major inventions were made on their campuses, inventions toward which they had contributed in equipment and faculty time, or inventions of apparent commercial value which the faculty member felt unable to manage in his own right. A second, more evolutionary, factor that directed university attention toward developing a general patent policy was the growth of university research laboratories. Industrial research and large university laboratories are phenomena of the twentieth century. As the university laboratories were doing more and more institutional research, a method for handling its results was demanded. The need for a general patent policy was particularly apparent in regard to research sponsored by industry.

1. *Patent Policy Developed for Single Spectacular Inventions*

One of the earliest spectacular campus inventions did not directly change university patent policy, but did cause the creation of the nonprofit Research Corporation. This organization, since its formation in 1912, has exercised a constant directing force in the development of methods for managing the results of university research.

---

[1] These fears were summarized in Chapter I.
[2] Ref. 42, p. 10.
[3] Ref. 28, p. 1319.

While an instructor in physical chemistry at the University of California in 1905, Frederick G. Cottrell [4] discovered the electrostatic precipitation method for recovering various products from flue gases. Mists and dusts were removed from the gases by applying the principle of attraction of oppositely-charged particles. The mist or dust was imparted a charge by passing through an intense electric field, as exists at the surface of a slender wire at high potential, and were collected on an oppositely-charged plate. The invention was first applied to recovery of sulfuric acid mists.[5]

During his summer vacations Dr. Cottrell, having formed the Western Precipitation Company with a few associates, began installing the electric precipitators. However, the business mushroomed so quickly that Cottrell found his commercial efforts were interfering with his desire to continue his scientific work. The University of California was offered the patents, but did not feel able to accept the assignment because of difficulties involved in such a commercial undertaking. Cottrell then offered the invention to the Smithsonian Institution, which in turn was instrumental in 1912 in organizing the independent, non-profit Research Corporation. Such distinguished scientists and engineers as Arthur D. Little, Elihu Thomson, T. Coleman Du Pont, and Charles A. Stone were among the founders.

Research Corporation, organized as a stock corporation whose shares may pay no dividends, set for itself two purposes: first, to build an efficient business organization to promote the precipitation process commercially, and to set an example for patent management in the public interest; and secondly, to apply all profits from this patent and others it might obtain to the support of further scientific research. As Dr. Cottrell himself has said, "The purpose in organizing Research Corporation was not merely to produce revenue for scientific research, but to act as a sort of laboratory of patent economics and to conduct an experiment in patent administration." [6]

Almost immediately heavy royalty returns from the Cottrell patents started to accumulate in a fund for financing university research. Grants were started in 1927 and among subsequent grants was one to M.I.T. to build the Van DeGraaff ten-million-volt electrostatic generator. Up to this year $1,250,000 had been distributed to some 52 universities all for research in pure science, and the Corporation[7] has recently announced that $2,500,000 more will be granted in the next five years. The University of California was one of the first universities to use the Research Corporation facilities in patent matters.

[4] Ref. 19, pp. 2–3.

[5] Ref. 11, p. 224. Dr. Cottrell described the trials and tribulations of the university inventorin a detailed and interesting article published in 1912. (Ref. 19.)

[6] Ref. 11, p. 224.

[7] Ref. 17, p. 34.

Two dramatic university discoveries in the twenties turned attention to the patent problem. They were preparation of Insulin at the University of Toronto (1921) and the conversion of ultra-violet energy into Vitamin D at the University of Wisconsin (1924). The University of Toronto has been authorized to own and exploit patents since 1906,[8] but not until the discovery made by Banting and Best were patents given particular consideration. The mechanism chosen to manage the Insulin patent, to control the quality of product sold by licenses and to apply royalties to further research was a special Insulin Committee within the University. In general, the University of Toronto has opposed taking patents for scientific discoveries in educational institutions,[9] but it has set up special committees to manage patents in two cases (in regard to Insulin and to an anti-silicosis invention) where control of public health discoveries appeared particularly necessary.

The Insulin patents[10] earned $500,000 gross for the University of Toronto in the year 1924 alone. The research men responsible have not participated in the income partly because of the prohibitive provision of the medical code of ethics and partly because of the belief at Toronto that giving a "cut" to the staff member degrades his research.

An entirely different method of patent management was undertaken at the University of Wisconsin. Dr. Harry Steenbock had developed methods of improving the antirachitic properties (Vitamin D) in food and drug products by irradiation with ultra-violet light from a quartz mercury lamp. Believing from a previous experience with patents on Vitamin A that the administrative machinery of the University was too cumbersome to handle patents effectively,[11] Dr. Steenbock was willing to hand over the patent rights to a non-profit organization formed in 1925 by interested Alumni under the name of the Wisconsin Alumni Research Foundation. The purpose of the Foundation was to commercialize the Steenbock and other patents and to build up from license royalties an endowment principal, the income from which could advance scientific research in any field at the University of Wisconsin. A control laboratory was established to fix standards of potency and to work out methods of test control for Vitamin D products, and some eight pharmaceutical firms were licensed to manufacture a standard product under the trade-name "Viosterol." By 1930, less than five years after organization, the Foundation had a gross income of about $100-per-day and had acquired several other patents, notably the iron-copper patent for the treatment of anemia, that have since proved profitable. The Foundation was already involved in the litigation, which has accompanied its financial success throughout 22 years of existence.[12] Patent

[8] Ref. 40, p. 100.

[9] Statement of Lorne Hutchinson, Exec. Secr. of the Insulin Committee, in ref 39, p. 100 and in ref. 15, pp. 330–332.

[10] Ref. 18, p. 60.

assignment to the Foundation by staff members was entirely voluntary, but such assignment was frequently resorted to by University inventors partly to avoid the business problems of patent development and partly to earn research funds for the University of Wisconsin. The inventor received at least 15 per cent of the net income. Prof. Steenbock [13] had received, up to 1940, $760,000 out of total royalties of $7,500,000 on the Vitamin D patents.

A third patent,[14] also in the field of public health, will be only mentioned in passing. Doctors George and Gladys Dick developed a scarlet fever antitoxin while associated with the John McCormick Institute for Infectious Diseases in Chicago. No patent application was made initially for a year after publication of the experimental results, but the officers of the United States Public Health Service notified the inventors that it lacked the legal powers to obtain the necessary quality control and recommended that the inventors assume responsibility for maintaining serum quality by means of a patent. The Scarlet Fever Committee was incorporated in 1925 with almost exactly the same functions and objectives as the Insulin Committee. The Dicks received no part of the patent royalties.

2. *Patent Policy Evolving From Growth of Institutional Research*

It would be untrue to say that educational institutions were not thinking about the patent problem after 1920. State universities, whose experiment stations were doing considerable applied research in cooperation with industry, were particularly concerned with the problem.

As early as 1913, the Engineering Experiment Station of the University of Illinois asserted that results of experimental work paid for from University funds belonged to the University and that the staff inventor might be required to patent his invention and assign it to the University. A committee of the Association of Land-Grant Colleges and Universities in 1922 found three land-grant colleges with definite patent policies.[15] The committee's report contained in substance the following specific suggestions:

(1) An institution has a right to inventions made by staff members in course of regular duties or at the institution's expense.

---

[11] Ref. 48, p. 1; ref. 44, pp. 21–27.

[12] Discussion of the legal difficulties into which the Wisconsin Foundation has fallen within the past five years will be reserved for later in this chapter. In the above historical development the early influence of the Wisconsin experience upon university patent policy generally is emphasized.

[13] Ref. 44, p. 27.

[14] Ref. 7, pp. 327–30.

[15] Ref. 6.

(2) There may be cases where both inventor and institution have equity in the patent, a case which must be decided on its own particular merits.
(3) Only patents with no commercial value should be dedicated to the public.
(4) Universities should give job of patent management to an outside organization, as the Research Corporation, to avoid complications detrimental to the University.

At Purdue University just at the end of this trial period before 1930 another Research Foundation [16] was established with wider objectives than those of the earlier Wisconsin Alumni Foundation. Patent administration was only incidental to its primary purpose: "To promote *educational* purposes by encouraging, fostering and conducting scientific investigations and industrial research." [17] Organized as the results of inventions of commercial value in the agricultural and engineering experiment stations of Purdue University, the Purdue Research Foundation actively sought out research contracts with industry. It intended that the research problem posed by an industrial sponsor should be used as "a teaching mechanism for intensively training creative students." [18] Although the Foundation was willing to accept assignment of patents on inventions made at the University without Foundation sponsorship, most of the patent problems have come from the sponsored or cooperative research. The Purdue Foundation may be contrasted with the Wisconsin Foundation both as to the source of its patents and as to its breadth of functions.

In obtaining a large volume of contracts for research of genuine scientific importance the Purdue Foundation has been highly successful. Research was done for industry at cost, but all patents were assigned to the Foundation and "small" royalties were charged. Throughout its existence the Foundation has kept its eye on the educational goal in evaluating the patent situation, but in spite of this apparent nonchalance toward patent profits it has been financially successful.

3. *Concurrent Experience in Non-Profit Research Organizations*

In the period before the first World War several non-profit research organizations had been organized and were gaining experience in patent management, experience which was being watched by university administrators. These organizations are divided according to their purposes into two groups: first, those which sponsor research on behalf of private industry, and second, those that do research primarily for the benefit of the general public. Universities have counterparts of both

[16] Ref. 44, app: pp. 19–20.
[17] Article II, Articles of Incorporation, Purdue Research Foundation.
[18] Letter of G. Stanley Meikle, Director of Purdue Research Foundation, to Mr. J. R. Killian, Jr., Nov. 9, 1943.

types of research. The Mellon Institute of Industrial Research,[19] organized in Pittsburgh about 1907 in loose affiliation with the University of Pittsburgh, does research on specific problems of immediate practical interest to industrial corporations. In keeping with its declared function to be a research service organization for industry, all inventions in the field of the investigation belong outright to the donor of the research funds. The individual inventor, who makes a discovery as a Mellon Institute fellow, is considered in the same legal position as the employee of an industrial laboratory.

Research Corporation exemplifies the organizations acting primarily for the public. The Bartol Research Foundation of The Franklin Institute and the Carnegie Institution of Washington, organizations similarly concerned with patent management in the public interest, were both functioning before 1910.[20] Fellows of the Bartol Foundation are required by a 1926 plan to assign all patent rights to the Foundation, but the inventor receives 40 per cent of the patent profits. In contrast the Carnegie Institution as early as 1923 took the stand "that any new and useful inventions or discoveries which may result from researches financed by the Institution shall be dedicated to public use." The Institution will not receive profit from patents, although it does use patents for purposes of control and commercial promotion.

4. *Summary to 1930*

At the beginning of the depression decade the patent problem was clearly drawn and several methods of handling the problem were being worked out. Two different factors had turned university attention to the patent problem: first, a steady growth of research sponsored cooperatively by industry demanded a generally applicable policy toward resulting patents, and, secondly, spectacular inventions on university campuses demanded immediate concern with patent policy. The majority of universities had tacitly taken the stand that they had no equity in patents of staff inventions not even those inventions made through use of the time and facilities of their institutions. A few had taken the reverse stand and required assignment to the university or at least submission to a decision by a university committee. There were available three types of experience for managing college patents:

(1) By an outside non-profit agency, as the Research Corporation.
(2) By a separate legal entity, loosely affiliated with the university, as the Wisconsin and Purdue Research Foundations.
(3) By a special university committee as the Toronto Insulin Committee.

---

[19] Ref. 32.

[20] Information summarized in this paragraph is taken from ref. 40, pp. 126–7.

The time was ripe for all universities conducting scientific research to formulate a definite patent policy.

B. *Experience since 1930*

Since the beginning of the thirties there has been an accelerated trend toward university development of patents on inventions made by staff members. Associated with this trend has been the growth of separate legal corporations affiliated with universities for the purpose of exploiting patents in their behalf. Dr. Palmer's 1934 study of university patent policy [21] found some 18 institutions developing patents; his current study,[22] as yet unpublished, shows nearly 200 universities and colleges with direct interest in patents. In at least 20 cases the separate research foundation manages patents for the university. Five other universities have a standing arrangement with Research Corporation for patent-management.

1. *Reason for a Definitive Patent Policy*

Several factors may be responsible for the stepped-up interest of universities in patents. First was economic necessity engendered by the Depression. Both endowment income and tax revenues were drying up; consequently, the almost unbelievable profits of the Steenbock and the Insulin patents were extremely appealing. The Wisconsin Alumni Research Foundation was able to turn over $325,000 to the University when the mother institution met hard times in 1933 and 1934.[23] Universities had, of course, considered the possibility of patent profits before, but they had usually spurned the proposition, believing that with the bad publicity that might accompany patent exploitation the university endowment might in the end actually suffer financially. This last fear, though still effective in the thirties, was overpowered by the compelling need for new sources of income. University administrators turned to working out methods of patent management that would not be detrimental to their public relations. The separate foundation was believed to be a partial solution.

A second factor was the growing belief, particularly in the state colleges, that where public funds spawned invention, the public (i.e. through the university) should receive the patent profits. Also, scientists were becoming aware of the advisability of patenting (as outlined in Chapter I) and, realizing their own limitations in the commercial field, they turned over their patents to the university or its agency for development, control, and profitable exploitation. Within a few years after 1930 many articles appearing in scientific journals, under such

[21] Ref. 40.

[22] Interview of this writer with Dr. Archie M. Palmer, Washington, D. C., April 11–12, 1947.

[23] Ref. 48, p. 7.

titles as "Society's Need for Patents to University Research Workers" and "Should Scientific Discoveries be Patented?",[24] discussed the pros and cons of university patents.

A final, and perhaps most important, factor in the development of a generally-applicable patent policy was the increase in the quantity of university research, particularly research done cooperatively with industry. A steady stream of research from which many patentable discoveries might result demanded that universities develop a generalized policy toward patents. Purdue and Ohio State faced up to this problem in forming their Research Foundations; a similar motivation existed for other engineering colleges performing research cooperatively with industry. Numerous universities, because of the huge amount of institutional research that they are conducting with industrial and governmental sponsorship as an aftermath of the war, are currently formulating a patent policy.

2. *Motivation for Formation of University Research Foundations*

Several reasons prompted formation of the Research Foundation as a legal entity separate from the university:

(1) Many restrictions on universities, particularly on state institutions, prevented them from contracting directly with industry and from managing and commercializing patents through their regular business offices.

(2) Fear that the tax-exemption of educational institutions might be revoked if they too-obviously engaged in patent exploitation and other commercial practices impelled universities to use the legal artifice of a separate corporation which they in fact controlled.

(3) The foundation scheme put to use for the university talents of alumni and other interested business men and industrialists in managing patents or research. This was originally the case in the Wisconsin Alumni Research Foundation and was particularly true in the formation of the Ohio State Research Foundation, of which such men as Charles F. Kettering are trustees.

3. *University Patents and the Recent Interpretation of the Anti-trust Laws*

The influence of the Wisconsin Foundation in hastening development of university patent policies and management procedures has already been shown. Another side of the picture appearing during the past decade has had an appreciably different influence on patent policy. Since about 1940 the Foundation has been under increased attack by the Anti-Trust Division of the Department of Justice for alleged prac-

[24] Ref. 28; ref. 29.

tices in the restraint of trade, and in 1944 a circuit court of appeals invalidated the Steenbock patents, holding that the restrictive methods used in commercializing them were "unwarranted and against the public interest." [25] The Wisconsin Foundation has fallen into some of the pitfalls that beset a university in the management of patents. Its practices through the resultant bad publicity have done great harm to both the University with which the Foundation is indirectly connected and to the cause of an effective and beneficial university patent program.

Assistant Attorney-General Wendell Berge [26] has extensively documented the case against the Wisconsin Alumni Research Foundation. His charges may be summarized as follows:

a. The Wisconsin Alumni Research Foundation "has been the vehicle for creating a domestic monopoly resulting in division of fields, price fixing, control of container size, and limitation of potency of vitamin products—as a result of which the public has been charged excessive and arbitrarily high prices." A licensee of the Foundation describes it as being "merciless in beating out competition."
b. It has suppressed use of competing Vitamin D processes by licensing arrangements and by threats of patent litigation.
c. It entered international cartels to divide the world into non-competitive areas.
d. In spite of its semi-independent nature the Alumni Foundation has been used in the interest of the Wisconsin milk interests to forbid use of Vitamin D irradiation for oleomargarine and to strengthen the concentration of the condensed milk industry in the State of Wisconsin.[27]
e. Its practices have been detrimental to the advancement of science in
   (1) its lack of interest in vitamin research unless a commercial advantage could be obtained,
   (2) its use of "its licensing scheme to discourage research by its licensees," and
   (3) its attempt to suppress scientific data and truthful advertising in order to maintain its monopoly position.

The financial success of the Steenbock patents is largely attributable to the willingness of pharmaceutical, chemical, and food companies to come under the Wisconsin price "umbrella," share the field of sales by

[25] The three U. S. licensees of the Toronto Insulin Committee were charged in 1941 with price fixing in violation of the Sherman Act and they paid large fines rather than assume the costs of litigation. Ref. 16, p. 112.

[26] Ref. 1, pp. 82–111. Mr. Berge's chapter on the vitamin monopoly is interesting reading and presents in detail the facts substantiating the above summary.

[27] Ref. 23, p. 497.

license arrangement, and pay the fairly reasonable royalty of 3 to 10 per cent on a large volume of sales.

The Wisconsin experience points up a dilemma that will face many universities: can maximum commercial profits be obtained and the public welfare preserved at the same time? The Wisconsin Foundation set out originally on the acceptable objective of effecting "rigid control through which it is possible to protect the public and prevent unscrupulous commercialism from capitalizing on the Steenbock discovery." [28] The fact that the control was perverted in order to make money and to serve special interests shall not obscure the worthiness of the goal; it only increases the necessity for finding the middle ground.

C. *Present Patent Practice in American Universities*

With a background understanding of the past motivation for patent policy, the reader may now wish to know what universities are actually doing about patents. Because of the dearth of organized information about patent policies of universities and colleges, the extensive table of the Appendix* was prepared to survey present patent practices.[29] Although the patent problem has been broken into its several facets to facilitate study, the essential unity of the total patent policy of a given university should be remembered. Only with difficulty can the patent policy of a university be pushed into "boxes" for comparison with other universities in the vertical columns of a table; thus unwarranted simplification must be guarded against.

1. *Policy toward Patent Exploitation as an Income Source*

The most significant differences in the patent policies of universities arise in their attitude toward income from university-controlled inventions. To quote a monograph now being prepared by the Department of Justice,[30] "the great majority of the institutions studied appear to be willing to use patents to produce income, and a substantial number, particularly those which have established foundations, are engaged in an active program of obtaining patents and administering them for profits." Dollar-and-cents data on the success of universities in obtaining patent profits are not available. Although Dean Potter of Purdue,[31] who has gained considerable experience in patent management, has written that "the income accruing to educational institutions from patents is insignificant," present data, although still scanty, in-

[28] Ref. 48, p. 4.

* The Appendix will follow Part II of this paper in the April issue of this JOURNAL.

[29] Note: All illustrative material used in this section (c) is taken from the Appendix, the sources of which are indicated completely there. Source references will not be given in this section for facts summarized in the Appendix.

[30] Ref. 44, p. 18.

[31] Ref. 27, p. 7.

dicate that the income is more substantial than he suggests. Although under these circumstances no definitive answer is possible, the indication is clear that several universities are receiving significant income from patents and that, as the present patents are brought into wider use and as more patents are gathered into the portfolios of universities, this income may be expected to increase.

Five universities, obviously a very small minority of all educational institutions, have expressed themselves as being vehemently opposed to profits from inventions. Fifteen years ago a faculty committee at the University of Pennsylvania adopted the resolution that "inventions or improvements made by [the University] . . . should not be restricted by the University, but should be announced to the world so that such benefits may be freely enjoyed by all, and *without pecuniary profit either to the University or to any one in its service.*" [32] A later statement of policy by the University of Pennsylvania permits patenting of discoveries for profit outside the field of public health. In 1943 the Board of Trustees of the University of Chicago took an uncompromising position against patents in stating: "The University will not profit financially from research by means of patents, royalties, or licensing agreements. Members of the staff will not be permitted to receive direct or indirect financial return from patents."[33]

Harvard, Yale, and Johns Hopkins Universities adopt approximately similar attitudes toward institutional patents: [34]

a. No patents are taken by the university except for dedication to the public. Harvard and Johns Hopkins believe it undesirable for either a university, or any agency associated in the public mind with its name, to hold patents.
b. Patenting is the private concern of the individual staff inventor, except that "no patents primarily concerned with therapeutics or public health may be taken out by a member of the University."

Some universities, including those above, consider medical patents in a category by themselves. Dr. Fishbein of the American Medical Association [35] finds two major objections to receiving profit from public health discoveries. First, patent competition among universities, such

[32] Ref. 40, p. 124; ref. 38, p. 55. The italics are supplied by this writer.

[33] Ref. 44, appendix, p. 3. This action by the Univ. of Chicago was called a "blow to the American patent system" by the American Patent Law Association.

[34] Ref. 44, pp. 18–19. The Harvard policy was laid down by faculty vote in 1934 after embarrassing difficulties with a patent on the Drinker respirator, devised by a member of the School of Public Health. (Ref. 8, p. 424.)

[35] Ref. 12, p. 1317. A thorough discussion of the pros and cons of medical patents was made by a 1939 Conference on Medical Patents called by the American Medical Association (refs. 7 and 8).

as undoubtedly existed among three or four over Vitamin D patents, is particularly undesirable in this field. Secondly, public relations of universities are susceptible to harm from charges of profiting from the misfortunes of the public. Besides the Vitamin D patents, the scarlet fever patents held by the Scarlet Fever Foundation and the patent on the copper-iron treatment for anemia held by the Wisconsin Foundation have been sources of friction. In spite of the objections to public health patents, many universities have used them to raise funds to finance further research. Toronto [36] holds the patents on preparation of Insulin; St. Louis, Theelin (the female sex hormone); Minnesota, Thyroxine (active element of thyroid gland used to treat goiter, cretinism, etc.); Wisconsin, Cincinnati, Columbia, and M.I.T., various Vitamin D patents. The objections of Dr. Fishbein are fundamental only if the public interest is injured by the end results of medical patents; otherwise, he is only pointing out difficulties encountered in administering a profit-making policy. The public interest can be served by university patent income in two very important ways:

a. The financial incentive to the scientific man and the protection of the manufacturer in the developmental stage can increase the chance that health discoveries will be developed to practical use.
b. The profits may be applied to further research to insure a steady flow of public-health discoveries.

Only the small minority of universities now consider public health patents improper sources of income. It is noteworthy that the few universities opposing patent profits are private institutions as contrasted with land-grant colleges which have led in developing patents for profit.

2. *Patent Ownership* [37]

Although many universities accept control of patents for profit, several variant philosophies in regard to ownership of the original patent rights are current. A large number of universities and colleges have no definite patent policy, although few universities doing appreciable scientific research have failed by now to give it consideration. Another small group of institutions, including notably the University of California, Syracuse University, and Louisiana State University, expressly leave all patent rights to the inventor, but nonetheless these universities do in special cases receive and administer patents. The University of Texas, in a recent change of policy, leaves all ownership rights to the

[36] In fairness to the University of Toronto it should be noted that it discourages patenting scientific discoveries of any type; control of Insulin quality was considered more important than profit from its sale. (Ref. 39, p. 100.)

[37] This discussion does not involve ownership of patents arising from research sponsored by outside corporations or government. These patents demand special consideration.

staff member, but requires payment to the university of a small percentage of net proceeds from the individually-developed patents.

A third and more sizable group of universities also follows the general policy that all patentable inventions belong to the staff member responsible, but they actively encourage assignment of the patents to the university or its agency for administration. The University of Wisconsin has gathered a valuable group of patents in its associated agency, the Alumni Research Foundation, on this voluntary basis. Columbia University, University of Minnesota, California Institute of Technology, and Ohio State University belong also in this group. The success of these universities in receiving voluntary patent assignment derives principally from two factors:

a. Inventors desire to avoid the business problems of patent commercialization.
b. They desire that their discoveries be brought to public use in such a manner as to provide funds for further university research.

The trend among universities is to adopt a fourth type of ownership policy, one which requires faculty members to assign inventions resulting from their "regular duties on University time and at University expense." [38] This policy approximates the one which governs the relation of the scientist or the engineer to his employer in industry. Land-grant colleges have long followed this policy since its initiation at the University of Illinois in 1913. Lehigh University and Massachusetts Institute of Technology led the way among private institutions by working out similar statements of policy in the early thirties. The survey made for the Appendix indicates at least fourteen universities clearly accept this philosophy of ownership. The required assignment is usually effected through voluntary acceptance of a faculty or board-of-trustees policy; in almost no case does a written contract exist between the university and faculty members, nor is any form of compulsion exercised. As an exception, the University of Florida requires reporting of inventions made by a faculty member in his own field of research. The University of Florida also has an out-of-the-ordinary policy in re-

[38] The breadth of interpretation that might be given this phase is evident. As Mr. Spencer of Northwestern Univ. Law School, ref. 42, p. 10, said, "Generally speaking, faculty members do not make a rigid distinction between their own time and university time,—a distinction which is very easily made in the business world,—because it is common practice for a university professor to mark examination papers during the evening and to devote certain daylight hours to some enterprise of his own." In general, inventions outside any specific university project are interpreted as the property of the faculty member involved. Contrary to the implication of Mr. Spencer's statement, the distinction between their own time and company time is difficult for scientists in industry to make, and usually all patents, at least in the technical field of the company, become company property. Of course, those universities requiring assignment of inventions by staff members may interpret the equities involved more liberally than industrial concerns.

gard to patents resulting from student theses. It is nearly universally considered elsewhere that a student through tuition payment receives rights to the results of his research; however, the University of Florida requires assignment of the patent to itself and then divides the amount awarded the inventors in such a manner that two-thirds goes to the faculty member directing the thesis research, and one-third to the graduate student.

The trend toward assertion of university rights to patents should be studied against the background of the accompanying trend toward institutional research centers, as contrasted with uncoordinated individual research efforts. Patent management is only part of the greater whole of research administration. The trend is associated with recognition that, although patents must name individuals as inventors, it is increasingly difficult to say that one person, or even several, are solely responsible for the results of institutional research.

3. Treatment of Inventions Resulting from Outside-sponsored Research

In the treatment of the results of so-called "cooperative" research, the engineering experiment stations of land-grant colleges early met the need for a patent policy of general application. The universities were receiving funds from industry in which the patent system played a prime economic part.

Special treatment is given in nearly all cases to patents developed from outside-sponsored research. Nearly universally the university and the sponsor make a patent agreement before initiation of the research program; that agreement may frequently modify the rights of staff members and graduate students to their inventions. For example, both Carnegie Institute of Technology and the University of Wisconsin, while generally leaving full patent rights with the staff inventor, require assignment of sponsored inventions to the Institute or University.

Varied degrees of preferential treatment are given to the research sponsor. In several cases the patents are given outright to the sponsor. Full assignment appears justified if the research undertaken by a university is only a continuation of research initiated earlier by the cooperating firm, if full costs including overhead and administrative cost are paid by the sponsor, or if further developmental work is necessary before commercial application. Full assignment also appeals to universities which try to avoid the administrative detail and legal complications of patent management. The tendency, however, apparently is to give only partial rights to the cooperating company. These "partial rights" take any of several forms:

a. Remission of license fees. The minimum concession to the sponsor is usually a free unrestricted license in the technical field of the sponsored investigation. Carnegie Institute of Technology remits the license fee until such remitted funds add to twice the

total contribution by the sponsor or ten times his contribution in the year the patent was issued, whichever sum is greater.

b. Opportunity for the sponsor to purchase full patent rights by payment of a special patents charge which is frequently about 20 per cent of the total costs of the original research program, including overhead and administrative costs. Lehigh University, the University of Florida, University of Michigan, University of Colorado and Texas A. & M. are applying variations of this policy.

c. An exclusive license to the invention for a restricted period of time or a particular use to permit the sponsor to recoup his expenditures. A great many universities, including M.I.T., use this method to encourage research sponsorship.

Purdue University has been unusually successful in encouraging industrial contributions for research without relinquishing extensive patent rights.

4. *Patent Administering Agencies*

Since formation of the Wisconsin Alumni Research Foundation in 1925, comparable foundations have been organized at some twenty other universities, most of which are state institutions. The principal reasons for their establishment were reviewed earlier in this chapter. The foundations vary in the control exercised over them by the associated university. The Wisconsin Foundation and, in a lesser degree, the Ohio State Research Foundation are independent organizations which nevertheless act primarily in the interest of the associated university. However, nearly all the foundations formed during the past decade have centralized control in the faculty, administration, or board of trustees of the university. For example, the Foundation at the University of Connecticut authorized by State law in 1945 was organized directly under the board of trustees.

The foundations vary also in the breadth of their functions. The majority are chiefly patent-administering agencies; but some, as those foundations at Purdue and Illinois Institute of Technology, also carry on research or act as the go-between for the university and the outside contracting party.

The same motivation as that for forming legally-separate research foundations has caused some universities to use other agencies dissociated from the university for patent management. For example, the Research Corporation of New York at present administers nearly all patents of five universities: namely, M.I.T., Columbia University, Princeton University, University of California, and University of Arizona. Some ten other colleges use the Research Corporation for managing isolated patents. The Research Corporation can never remove all problems from the shoulders of the university; in fact in many

cases involving litigation or licensing policy it will consult the university concerned.[39]

In sharp contrast to the research foundation over which the university has only the slightest indirect control (as the Wisconsin Foundation) there are many educational institutions which control their patents directly. Some university administrators argue that an educational institution cannot and shall not shunt off its patent problems. Dr. Hutchison, executive secretary of the Insulin Committee, through which the University of Toronto has managed directly those valuable patents, has said cogently: [40]

> A non-commercial, scientific, academic organization has a very real responsibility to the public . . . in deciding in any instance as to whether or not a discovery or invention shall be patented. . . . After making that decision . . . it is the duty of the scientific institution itself . . . to control the outcome [of the decision to patent, assign, or license], and if it makes mistakes try to correct them, or at least to take responsibility for them, and not foist them off on some other organization.

The observed trend toward formation of legally-separate research foundations should be explained more as an attempt to gain legal flexibility in handling patents, rather than as one to avoid responsibility for patent management. Whether patents are managed through a research foundation or directly by the university administration much the same as other property, Dr. Hutchison's thesis apparently is now widely accepted, and very few research foundations exercise the independence from the parent university that the Wisconsin Alumni Foundation has enjoyed. The foundations formed in more recent years, as noted above, have been organized under the tight control of the university. It should be recognized that, in tying the research foundation closely to the university by means of officers acting in dual capacities, the legal insulation from litigation difficulties enjoyed by the University of Wisconsin is lost. Any court could doubtlessly break through the legal artifice of the separate corporation and impose its fine or other penalty directly on the university itself. Administration by an entirely dissociated organization, as Research Corporation, is in this respect preferable.

Besides the University of Toronto, St. Louis University has had considerable experience in direct management of patents. An administrative committee in the School of Medicine, the Committee on Grants for Research, has administered the Doisy patents for preparation of Theelin, the female sex hormone, since 1930.

5. *Reward to the Inventor*

With few exceptions the inventor shares in the profits from his discovery after he assigns the patent to the University or the associated

[39] From this writer's talk with Dr. Joseph W. Barker, President of Research Corporation, April 10, 1947.

[40] Ref. 39, pp. 101–102.

patent-administering agency. This policy of inventor reward, quite different from the practice in the research laboratories of large industrial concerns, is specifically supported in the patent-policy statement of Virginia Polytechnic Institute:[41]

> The research worker must have displayed more than routine zeal in the attack of a problem and . . . he must have drawn on his own mental equipment and energy to an extraordinary extent [to make a valuable invention]. . . . Employees of large corporations may reasonably expect that their inventions, even if assigned under contract to their employers, will result in increased salary recognition. . . . It is more difficult to secure such recognition in academic circles and it is believed preferable [at V.P.I.] to allow the research worker to retain a direct interest in any inventions resulting from his work.

The University of Toronto and St. Louis University are exceptions in giving the inventor no reward. Their patents have been concerned with public health, and in keeping with the code of ethics of the medical profession the doctors making the discoveries have not wished to profit directly. Actually they have received their reward in liberal grants for further research and in improved laboratory facilities.

V.P.I. as well as Lehigh University and Wittenberg College gives 50 per cent of net profits to the inventor. They believe that equity in the university patents is shared equally by the inventor and the university.

The Research Corporation gives 7 per cent of *gross* receipts to the inventor. Its philosophy is that while some reward to the inventor is necessary, its prime purpose should be to earn profits for the furtherance of research, distributed by fixed proportion (usually about 50 per cent) to the university from which the patent came and by special grants-in-aid for particular research projects.

The great majority of colleges allot 15 to 25 per cent of *net* patent profits to the inventor. Many universities, while fixing a minimum percentage, leave the exact proportion of receipts payable to the inventor to the discretion of the board of trustees or other administering agency. Although the fixed stipend protects the university administrator from charges of favoritism and political manipulation in distributing patent profits and is generally more easily administered, the flexible rate of inventor reward has one great advantage in recognizing the varying classes of patents that a university may administer and the varying amount of equity which the inventor may have in the patent rights. As universities gain more experience in patent matters, they may well investigate whether more of those patents toward which they have contributed but little might be assigned to the university if they gave a larger proportional inventor reward in such cases.

6. *Licensing Policy*

Little information is available about the licensing policy of universities even though this policy is of paramount importance in the effect

[41] Ref. 44, App., p. 24.

of university patents on the public welfare. In general universities prefer to give only non-exclusive licenses, a policy followed by Research Corporation for 35 years. The complex system of restrictions applied to the Wisconsin Vitamin D licenses has no parallel in the policy of other universities. However, many educational institutions are willing to give exclusive licenses in special cases, such as the following:

a. Cases where only an exclusive license will bring the invention into public use. The Texas A. & M. Research Foundation on occasion grants an exclusive license with a precautionary provision for diligence by the licensee in using the invention.
b. Cases where quality control of the manufactured products is best assured by licensing one or only a few manufacturers. St. Louis University gave an exclusive license for a period of a year and a half on its Theelin patents, including in the license agreement provisions designed to control quality.

In the problem of how to license the patents it owns, the university faces a fundamental dilemma between a desire for profits from patent exploitation and a wish to serve the public in a competitive economy. Profits can be maximized only by restricting the number of licenses given. A middle-of-the-road policy may be found, but the dilemma will remain.

William H. Nicholls of Iowa State College [42] after considering this dilemma in specific reference to the Steenbock patents argues:

> The only solution *from the social viewpoint*, therefore, is probably to patent the results of technical research in public institutions [but only in order to be in position to prevent their misuse] and grant their use free or on reasonable terms to all who find them technically and economically feasible and will meet the desired standards of quality.

He defines "reasonable terms" as license fees from which the justifiable cost of quality maintenance could be recouped; in other words, all net income from patents is "dirty money." He would not accept the above-mentioned cases for exclusive licenses as being justifiable. Mr. Nicholls is not entirely sound in supporting his argument by an appeal to social ends. Judicious use of exclusive licensing may promote the public welfare in very specific ways: by bringing more inventions into public use, by effecting better quality control of the manufactured product, and by supporting further university research. Mr. Nicholls argues that exclusive licensing can be used validly to bring patents into use only if the firms in the given industry are small and financially weak and if free competition exists to protect the producer and consumer. Actually, even old well-established companies frequently will not develop patents without some degree of patent protection. That the university has a difficult problem in balancing the diverse factors in the interest of the public is agreed to by all.

[42] Ref. 23, p. 498.

Also, in attempting to evaluate university licensing policy by an absolute criterion of its effect on competition, Mr. Nicholls overlooks the relative criterion which would compare the effect on competition of a university patent with the effect of an industrially-controlled patent on the same invention. There is little doubt that a licensing policy may be developed to yield income to universities without imposing the restrictions on competition inherent in an industrially-owned patent.

The one instance in which restrictive licensing has been used extensively to the detriment of the public good is a case in which a research foundation had a free hand to manage the patents as it wished. The present tendency to keep control of all patent-administering agencies tightly in the hands of the university is doubtlessly a trend in the right direction. The public consciousness of the university is greater than that of an independent research foundation, such as the Wisconsin Foundation. The university itself, an increasing number of patent administrators appear to believe, must assume direct responsibility for making and executing its patent policy. This statement does not imply, however, that a university may not use other institutions, such as Research Corporation, with objectives proved by long years of experience to be similar to its own, to perform the actual mechanics of patent management.

(To be continued.)

Rising Fertilizer Tonnage.—An estimate of 15 million tons of commercial fertilizers used in 1946 sets a new high record for the United States. A table prepared by A. L. Mehring of the Department of Agriculture shows that 1942 was the first year in which fertilizer tonnage was more than 10 million tons, and 1910 was the year when fertilizer passed the 5 million ton mark.

The rapid wartime increase in fertilizers was in contrast to the record in World War I. In 1914, the record shows a consumption of more than 7 million tons, which dropped to a little more than 5 million in 1916, and did not pass the 1914 figure again until 1920. In 1921 the figure dropped below 5 million. A year by year increase followed until 1930 with nearly 8½ million tons, followed by a "depression" slump to about 4½ million tons in 1932. Since then the trend has been sharply upward. Use of fertilizer more than doubled in the 10 years from 1936 to 1946, says Mehring.

R. H. O.

Sound Waves Inspect Metal. (*Compressed Air Magazine*, Vol. 52, No. 9.)—During the war, German technicians developed a supersonic method of locating bubbles in sheet metal too minute for detection by X-ray or magnetic processes. A report on the device prepared by Dr. G. J. Thiessen, a member of the National Research Council at Ottawa, Canada, states that the method tests sheet metal for imperfections by comparing the energy of supersonic vibrations before and after passage through the metal. Anything greater than the normal drop in transmission indicates the presence of a flaw. Experiments have shown that a bubble only one-hundred thousandth of a millimeter in diameter can cause a hundredfold drop in energy. Slag- or rust-filled bubbles do not cause a drop of like magnitude but, even so, are readily discovered.

The supersonic vibrations are generated by a 15-watt quartz-crystal sender that operates at 1,000 kilocycles and "wobbles" the frequency 100 cycles per second to prevent standing waves in the metal undergoing test as well as between the sender and receiver. The vibrations are transmitted by a metal block to a narrow chamber of running water which, in turn, passes them on by making contact with one edge of the test metal. A similar arrangement picks up the vibrations at the other edge and forwards them to the receiver. The sending and receiving units are attached to opposite ends of a long pair of tongs that straddle the sheet, move along its edges, and are kept in contact therewith by hand pressure or by springs in the tong handles.

Full details about this and other methods of inspecting metals are contained in a report, entitled "The Non-Destructive Testing of Materials and X-Ray Protection Methods," which can be obtained in photostat, microfilm ($1), or mimeographed form (25 cents) from the Office of Technical Services, Department of Commerce, Washington 25, D. C.

R. H. O.

CONSTRUCTION OF ALIGNMENT NOMOGRAM FROM EMPIRICAL DATA.

BY

LO-HO,

Tangshan Engineering College, Tangshan, North China.

INTRODUCTION.

The equations that can be exactly represented by nomograms are only a few special cases of the vast number of functional relations which will never cease to appear in different forms while man's inquiry into the secrets of nature is going on. Modern scientific literature is full of such functional relations of which many have not been concisely and accurately represented by suitable equations and are left in the primitive form of tabulation of values. Whether in the form of an equation or in the form of a tabulation of values, they are functional relations for which nomographic representation is desirable. It is the purpose of this paper to describe a general method of constructing alignment nomograms from a tabulation of values. The resulting nomograms are necessarily approximations. So long as the approximations are close enough for practical purposes, it matters little whether they are theoretically correct or not. All graphical methods are based upon measurements and they can never be perfectly correct.

METHODS OF APPROACH.

Functional relation between three variables involved in a practical problem is generally continuous and single-valued. It may be represented by a family of curves showing the relation between any two variables while the third one takes different constant values, or by a surface in space.

When the family of curves can be transformed all at the same time into straight lines, the relation can be represented by a nomogram. Wertheimer [1] gave a graphical method of transformation for the possible case.

When the relation is considered as a continuous and smooth surface, a more general method known as surface fitting may be used to construct the nomogram with higher accuracy.

SURFACE FITTING.

In Fig. 1 is shown a surface representing a function of two variables

$$r = f(t, u)$$

[1] Albert Wertheimer, "The Graphical Transformation of Curves into Straight Lines and the Construction of Alignment Nomogram," JOURNAL OF FRANKLIN INSTITUTE, March, 1935.

or experimental data of a bi-variate relation according to the usual method of coordinate geometry. The three ordinate distances of any point on the surface represent the values of t, u and r that satisfy the condition of the relation.

When an equation can be represented by an alignment nomogram, its surface is said to be nomographic.

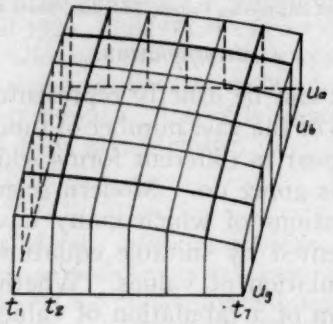


FIG. 1.

By surface-fitting is meant the process of substituting a surface of known character or more particularly a nomographic surface for a surface defined by a given equation or a given table of values. The object is to find a nomogram giving a surface which approaches everywhere or coincides in as many points as possible with the original surface. By a graphical method to be described, it is possible to construct a nomographic surface which possesses in common with the original surface four sections determined by two values of each of the variables t and u.

When two smooth surfaces possess four common sections such as at t_2, t_6, u_b and u_f shown in Fig. 1, it is very likely that they approach each other very closely at any other part within the region. As will be shown later, it is still possible to manipulate the nomographic surface so that an additional point common to the two surfaces may be created. The error of this method of approximation may be computed; and examples of many practical problems indicate that the errors are small.

METHOD OF CONSTRUCTION.

In actual construction the work may be started with a table giving the values of the function corresponding to given values of the variables as shown in Table I, where the symbol d_2 stands for the value of r at $u = d$ and $t = t_2$. Most of the values not required in the construction are not given. On a straight line r as shown in Fig. 2 a natural (or functional) scale is marked covering the values of r from the minimum to the maximum. Then choose any two suitable points to be used as

TABLE I.

u \ t	t_1	t_2	t_3	t_4	t_5	t_6	t_7
u_a	a_1	a_2				a_6	
u_b	b_1	b_2	b_3	b_4	b_5	b_6	b_7
u_c		c_2				c_6	
u_d		d_2				d_6	
u_e		e_2				e_6	
u_f	f_1	f_2	f_3	f_4	f_5	f_6	f_7
u_g		g_2				g_6	g_7

the graduation points t_2 and t_6; locate the graduation points a_2 and a_6 on the r scale; and draw two lines from t_2 and t_6 and passing through a_2 and a_6 respectively. The meeting point of these two lines will be used as the graduation point u_a on the u scale. Similarly the values b_2 and b_6 are used to determine u_b and so forth until all the graduation points of the u scale are located and a smooth curve is drawn through them.

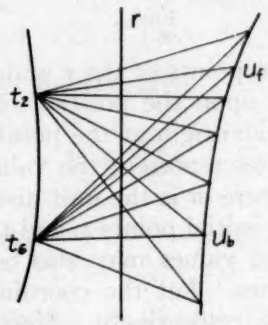


FIG. 2.

For the construction of the t scale, two suitable points such as u_b and u_f of the u scale may be used as the common points of projection. In this case, corresponding values of r in the rows u_b and u_f are used in locating the required points of the t scale.

Fig. 2, when used as an alignment nomogram, will give the correct values of the function at $t = t_2$, $t = t_6$, $u = u_b$ and $u = u_f$.

CONSTRUCTION OF SCALES BY RECTANGULAR COORDINATES.

It is still possible to plot the scales of this kind of nomogram by rectangular coordinates When the points t_2 and t_6 or u_b and u_f are too

near to the r scale, it becomes necessary to prolong two short lines to have an intersection at a great distance away. This means inaccurate location of the required points. In such case, the method of plotting with computed coordinates may be used. It is only the method of computation that needs some explanation.

The r scale may be assumed as the Y-axis and any line perpendicular to it may be taken as the X-axis as shown in Fig. 3. Then the X-ordi-

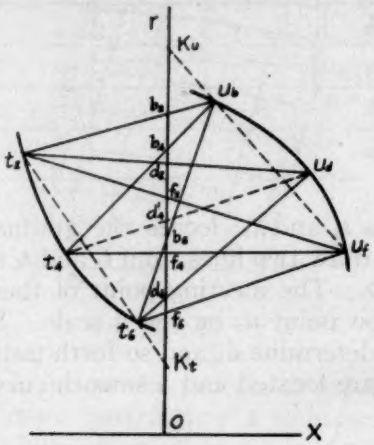


FIG. 3.

nates of all the graduation points of the r scale are equal to zero; and their Y-ordinates depend upon the position of the origin. It is perfectly general to assume the origin at the point where the value of r is zero. Then the Y-ordinates represent the values of r according to the scale equation $Y = mr$ where m is the unit distance of the scale.

The coordinates of the initial points t_2 and t_6 may be measured from the figure. Nearest round values may also be assumed and used instead of the measured ones. Let the coordinates of t_2, t_6 and u_b be X_2, Y_2, X_6, Y_6, X_b and Y_b respectively. Since the Y-ordinates of the points of intersection of the t_2u_b and t_6u_b with the r scale are mb_2 and mb_6 respectively, their equations may be written as

$$\begin{vmatrix} X_2 & Y_2 & 1 \\ X_b & Y_b & 1 \\ 0 & mb_2 & 1 \end{vmatrix} = 0 \quad \text{and} \quad \begin{vmatrix} X_6 & Y_6 & 1 \\ X_b & Y_b & 1 \\ 0 & mb_6 & 1 \end{vmatrix} = 0$$

which give the values of X_b and Y_b as follows:

$$X_b = \frac{X_2X_6m(b_2 - b_6)}{X_2(Y_6 - mb_6) - X_6(Y_2 - mb_2)}, \tag{1}$$

$$Y_b = \frac{X_2mb_2(Y_6 - mb_6) - X_6mb_6(Y_2 - mb_2)}{X_2(Y_6 - mb_6) - X_6(Y_2 - mb)_2}. \tag{2}$$

These two equations may be used as the general formulae for computing the coordinates of the graduation points of the u scale. Similar equations may be derived for computing the coordinates of the graduation points of the t scale in terms of those of u_b and u_f and the values of r.

ERROR OF APPROXIMATION.

The continuous lines shown in Fig. 3 are those actually used in the construction of the scales of t and u. Their intersections with the r scale give the values of r in Table I. But the value d_4' determined by the dash line t_4u_d may not be the same as d_4, the true value of r at t_4 and u_d. The difference

$$D_4 = d_4 - d_4' \tag{3}$$

represents the error of approximation of the nomogram which may be found as follows:

The coordinates X_b and Y_b, given by (1) and (2), may be substituted into the equation

$$\begin{vmatrix} X_4 & Y_4 & 1 \\ X_b & Y_b & 1 \\ 0 & mb_4 & 1 \end{vmatrix} = 0$$

of the line t_4u_b. The resulting equation will be

$$X_4\{X_6(Y_2 - mb_2)(b_4 - b_6) - X_2(Y_6 - mb_6)(b_4 - b_2)\} + Y_4X_2X_6(b_6 - b_2) - mX_2X_6b_4(b_6 - b_2) = 0.$$

Similarly, the lines t_4u_f and t_4u_d give

$$X_4\{X_6(Y_2 - mf_2)(f_4 - f_6) - X_2(Y_6 - mf_2)(f_4 - f_2)\} + Y_4X_2X_6(f_6 - f_2) - mX_2X_6f_4(f_6 - f_2) = 0$$

and

$$X_4\{X_6(Y_2 - md_2)(d_4' - d_6) - X_2(Y_6 - md_2)(d_4' - d_2)\} + Y_4X_2X_6(d_6 - d_2) - mX_2X_6d_4'(d_6 - d_2) = 0,$$

respectively. The substitution of X_4 and Y_4 found from any two of these three equations into the third one will give, on simplification, the following determinant:

$$\begin{vmatrix} \dfrac{b_6 - b_2}{b_4 - b_2} & b_4\dfrac{b_6 - b_2}{b_4 - b_2} & \dfrac{Y_2X_6 - Y_6X_2}{X_6 - X_2} - mb_6 \\ \dfrac{d_6 - d_2}{d_4' - d_2} & d_4'\dfrac{d_6 - d_2}{d_4' - d_2} & \dfrac{Y_2X_6 - Y_6X_2}{X_6 - X_2} - md_6 \\ \dfrac{f_6 - f_2}{f_4 - f_2} & f_4\dfrac{f_6 - f_2}{f_4 - f_2} & \dfrac{Y_2X_6 - Y_6X_2}{X_6 - X_2} - mf_6 \end{vmatrix} = 0.$$

Substituting

$$d_4' = d_4 - D_4$$

and

$$mK_t = \frac{Y_2X_6 - Y_6X_2}{X_6 - X_2} \tag{4}$$

into the above determinant and solving for D_4, we have

$$D_4 = \frac{\begin{vmatrix} b_6 - b_2 & b_4(b_6 - b_2) & (K_t - b_6)(b_4 - b_2) \\ d_6 - d_2 & d_4(d_6 - d_2) & (K_t - d_6)(d_4 - d_2) \\ f_6 - f_2 & f_4(f_6 - f_2) & (K_t - f_6)(f_4 - f_2) \end{vmatrix}}{\begin{vmatrix} b_6 - b_2 & b_4(b_6 - b_2) & (K_t - b_6)(b_4 - b_2) \\ 0 & (d_6 - d_2) & (K_t - d_6) \\ f_6 - f_2 & f_4(f_6 - f_2) & (K_t - f_6)(f_4 - f_2) \end{vmatrix}} \tag{5}$$

which gives the error of the nomogram for any combination of the values of t and u.

It can be shown that the value of K_t given by (4) is equal to the scale reading on the r scale at its point of intersection with the line t_2t_6. Since the points t_2 and t_6 on the t scale are arbitrarily chosen at the beginning of the construction, they may be made such that the corresponding value of K_t fulfils the condition that $D_4 = 0$. This gives

$$K_t = \frac{\begin{vmatrix} b_6 - b_2 & b_4(b_6 - b_2) & b_6(b_4 - b_2) \\ d_6 - d_2 & d_4(d_6 - d_2) & d_6(d_4 - d_2) \\ f_6 - f_2 & f_4(f_6 - f_2) & f_6(f_4 - f_2) \end{vmatrix}}{\begin{vmatrix} b_6 - b_2 & b_4(b_6 - b_2) & b_4 - b_2 \\ d_6 - d_2 & d_4(d_6 - d_2) & d_4 - d_2 \\ f_6 - f_2 & f_4(f_6 - f_2) & f_4 - f_2 \end{vmatrix}}. \tag{6}$$

Similarly the other value K_u at the intersection of the line u_4u_f with the r scale is given by

$$K_u = \frac{\begin{vmatrix} b_2 - f_2 & d_2(b_2 - f_2) & b_2(d_2 - f_2) \\ b_4 - f_4 & d_4(b_4 - f_4) & b_4(d_4 - f_4) \\ b_6 - f_6 & d_6(b_6 - f_6) & b_6(d_6 - f_6) \end{vmatrix}}{\begin{vmatrix} b_2 - f_2 & d_2(b_2 - f_2) & d_2 - f_2 \\ b_4 - f_4 & d_4(b_4 - f_4) & d_4 - f_4 \\ b_6 - f_6 & d_6(b_6 - f_6) & d_6 - f_6 \end{vmatrix}} \tag{7}$$

and other equations involving the same set of data in an arrangement with respect to the variable u may be obtained.

When the value of K_t used in the actual construction is based upon $D_4 = 0$, the resulting nomogram will give a surface that has an additional point in common with the original surface at $t = t_4$, $u = u_4$. Actual examples indicate that this extra point of contact greatly reduces the error of approximation in the surrounding area.

EQUATION FOR DIRECT CALCULATION.

The expression for the value of d_4' given by the nomogram may be obtained from the original determinant as follows:

$$d_4' = \frac{\begin{vmatrix} b_6 - b_2 & b_4(b_6 - b_2) & (K_t - b_6)(b_2 - b_4) \\ d_6 - d_2 & 0 & (K_t - d_6)(d_2) \\ f_6 - f_2 & f_4(f_6 - f_2) & (K_t - f_6)(f_2 - f_4) \end{vmatrix}}{\begin{vmatrix} b_6 - b_2 & b_4(b_6 - b_2) & (K_t - b_6)(b_4 - b_2) \\ 0 & (d_6 - d_2) & (K_t - d_6) \\ f_6 - f_2 & f_4(f_6 - f_2) & (K_t - f_6)(f_4 - f_2) \end{vmatrix}}, \tag{8}$$

which may be used to compute what are theoretically given by the nomogram. As in the construction of the nomogram, only the data on the four specific sections are used in the calculation.

TEST OF ACCURACY.

As it is desirable to know before the start of the actual construction how closely the given surface may be approximated by a nonogram, an equation for computing the error but without the presence of K_t may be developed under an assumption which is reasonable and also desirable. If it is assumed that the initial points t_2 and t_6 are equidistant from the r scale, we will have

$$X_6 - X_2 = 0,$$
$$K_t = \text{infinity},$$

and

$$\frac{1}{K_t} = 0.$$

This reduces (5) to

$$D_4 = \frac{\begin{vmatrix} b_6 - b_2 & b_4(b_6 - b_2) & b_4 - b_2 \\ d_6 - d_2 & d_4(d_6 - d_2) & d_4 - d_2 \\ f_6 - f_2 & f_4(f_6 - f_2) & f_4 - f_2 \end{vmatrix}}{\begin{vmatrix} b_6 - b_2 & b_4(b_6 - b_2) & b_4 - b_2 \\ 0 & (d_6 - d_2) & 1 \\ f_6 - f_2 & f_4(f_6 - f_2) & f_4 - f_2 \end{vmatrix}}, \tag{9}$$

which may be used to compute directly from the data the error corresponding to d_4. It is expected that the central point, the four corner points and the four middle points of the four edges should have large errors, if not the largest, for they are located far away from the lines of coincidence. The errors at these points may be computed with (9) and used as guidance in locating the position of the point of contact in order to determine the value of K_t.

INCREASE OF ACCURACY BY SUB-DIVISION OF THE REGION.

Obviously the efficiency of the four lines of coincidence in holding together the original surface and that of the nomogram will be decreased when

(a) the original surface is undulating or irregular, and
(b) the region to be covered is too large.

In either case, when the calculated error is larger than allowable, we may divide the given region into several smaller ones as shown in Fig. 4, and construct separate nomograms for the individual sub-regions. Even in that case it is sometimes still possible to arrange the diagrams compactly in one plate as shown in Fig. 5.

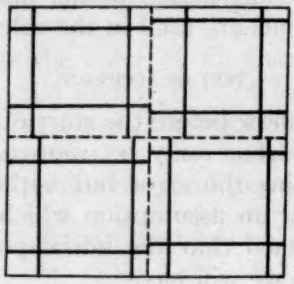


FIG. 4.

PRACTICAL CONSTRUCTION.

The final object in this method of construction is also a nomogram suitable for practical use; and the question of shape and dimension is still the main point of consideration in the actual process of construction.

As the error of approximation depends on the spacing and the location of the four lines of coincidence, and such lines must be assumed before the error can be computed, the designer has to make trials at the beginning. Examples indicate that the first trial calculation may be made with these lines somewhere at the outer eighth points as shown in Fig. 6 and the point of contact at center will give the value of K_1. Then the errors at other points may be calculated, and the accuracy of approximation of the given function by the proposed nomogram may be determined.

When the required accuracy cannot be reached by a single nomogram, further trials may be made with

(a) a functional transformation of the data as illustrated in Example 6 (see below), or
(b) a sub-division of the region as shown in Fig. 4 and illustrated in Fig. 5.

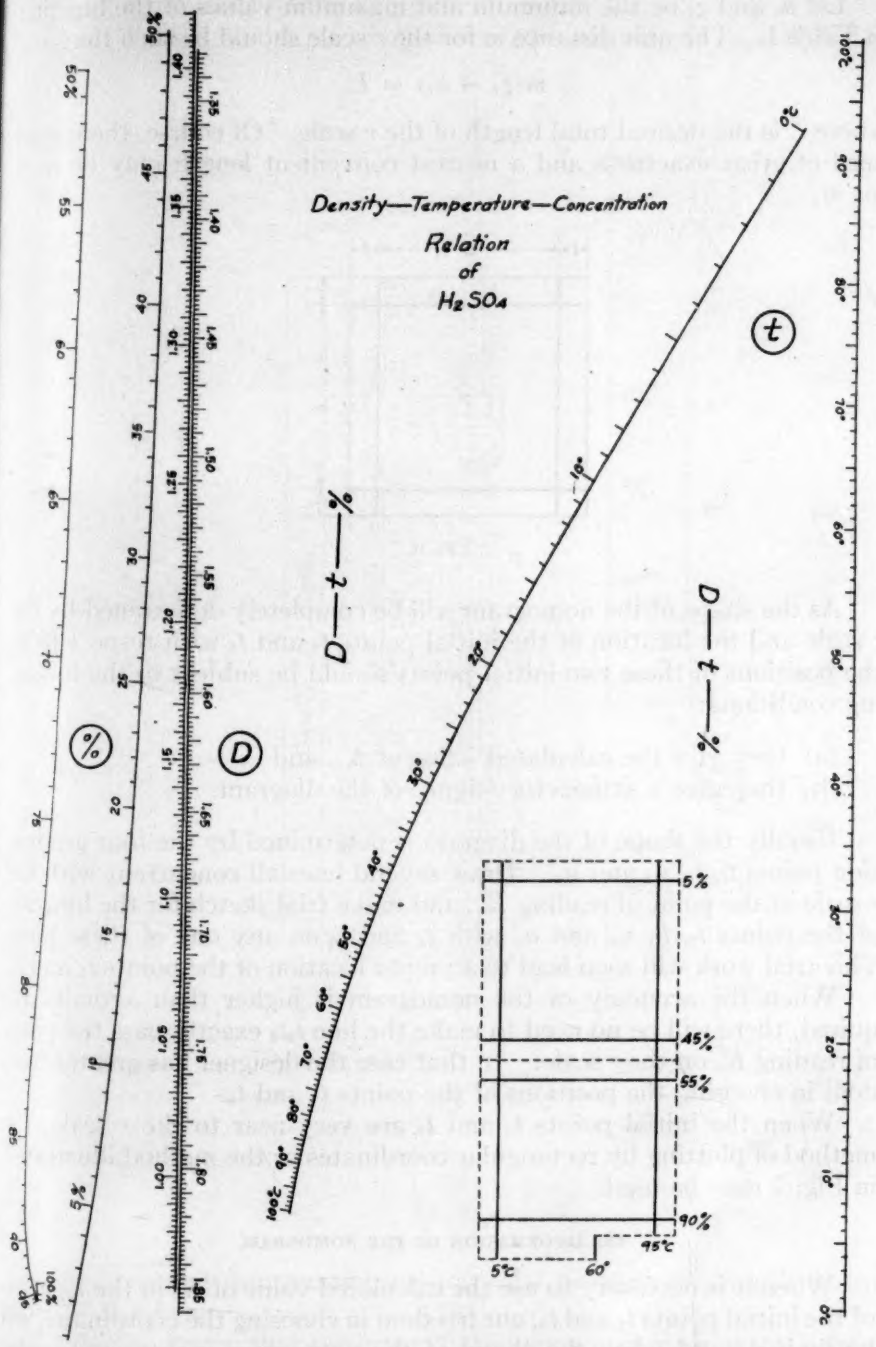

Density—Temperature—Concentration
Relation
of
H_2SO_4
D—t—%
%—t—D
%
D
t
5%
45%
55%
90%
5°C
60°
95°C

FIG. 5.

Let a_1 and g_7 be the minimum and maximum values of the function in Table I. The unit distance m for the r scale should be such that

$$m(g_7 - a_1) = L,$$

where L is the desired total length of the r scale. Of course, there is no need of great exactness and a nearest convenient length may be used for m.

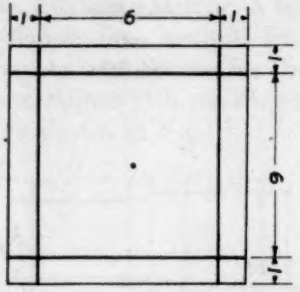


FIG. 6.

As the shape of the nomogram will be completely determined by the r scale and the location of the initial points t_2 and t_6 with respect to it, the positions of these two initial points should be subject to the following conditions:

(a) they give the calculated value of K_t, and
(b) they give a satisfactory figure of the diagram.

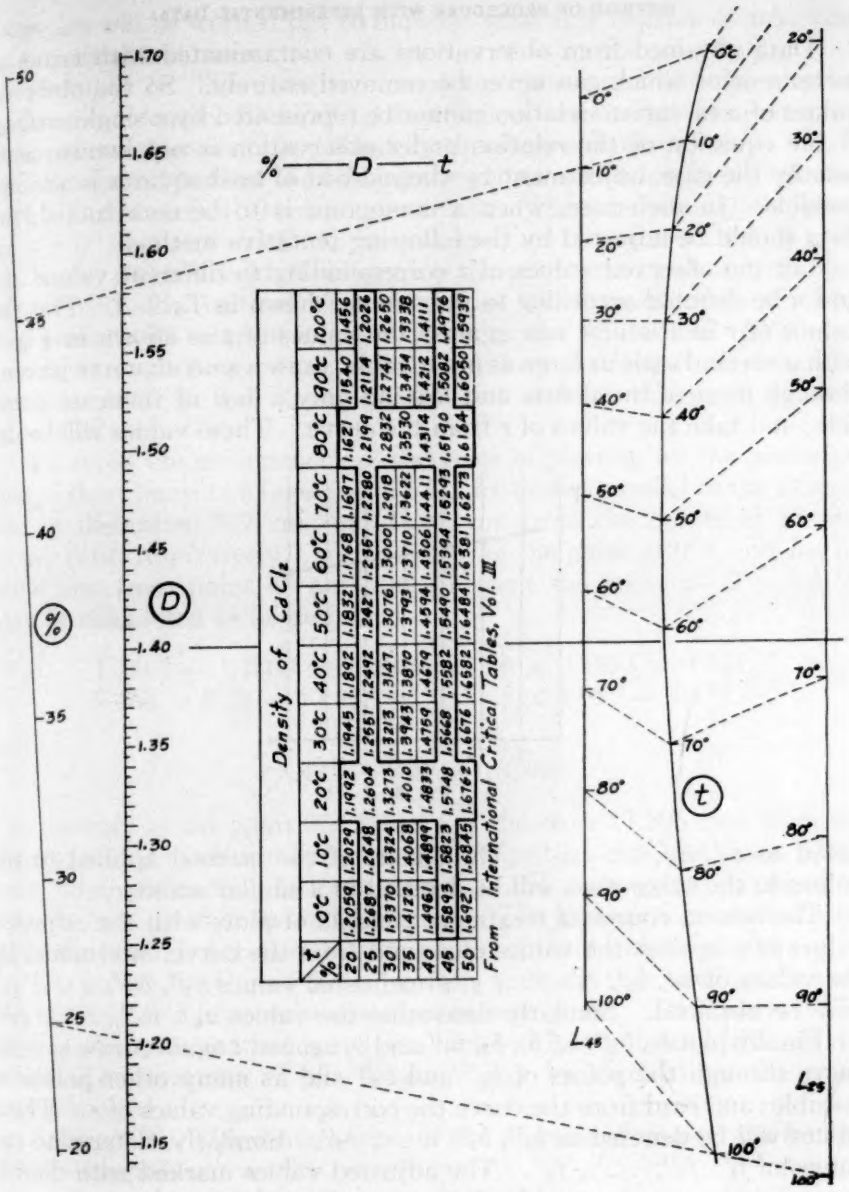
Usually the shape of the diagram is determined by the four graduation points t_1, t_7, u_a and u_g. Draw several lines all concurrent with the r scale at the point of reading K_t; and make trial sketch for the location of the points t_1, t_7, u_a and u_g with t_2 and t_6 on any one of these lines. This trial work will soon lead to a proper location of the points t_2 and t_6.

When the accuracy of the nomogram is higher than actually required, there will be no need to make the line t_2t_6 exactly pass the point of reading K_t on the r scale. In that case the designer has greater freedom in choosing the positions of the points t_2 and t_6.

When the initial points t_2 and t_6 are very near to the r scale, the method of plotting by rectangular coordinates or the method illustrated in Fig. 7 may be used.

TRANSFORMATION OF THE NOMOGRAM.

When it is necessary to use the calculated value of K_t in the location of the initial points t_2 and t_6, our freedom in choosing the coordinates will be limited; and when the shape of the nomogram constructed under this restriction is undesirable, the usual method of transformation based



% \ t	0°C	10°C	20°C	30°C	40°C	50°C	60°C	70°C	80°C	90°C	100°C
20	1.2059	1.2029	1.1992	1.1945	1.1892	1.1832	1.1768	1.1697	1.1621	1.1540	1.1456
25	1.2687	1.2648	1.2604	1.2551	1.2492	1.2427	1.2357	1.2280	1.2199	1.2114	1.2026
30	1.3370	1.3324	1.3273	1.3213	1.3147	1.3076	1.3000	1.2918	1.2832	1.2741	1.2650
35	1.4122	1.4068	1.4010	1.3943	1.3870	1.3793	1.3710	1.3622	1.3530	1.3434	1.3338
40	1.4961	1.4899	1.4833	1.4759	1.4679	1.4594	1.4506	1.4411	1.4314	1.4212	1.4111
45	1.5893	1.5823	1.5748	1.5668	1.5582	1.5490	1.5394	1.5293	1.5190	1.5082	1.4976
50	1.6921	1.6845	1.6762	1.6676	1.6582	1.6484	1.6381	1.6273	1.6164	1.6050	1.5939

FIG. 7.

on mathematical computation or graphical projection can still be applied in this case.

It can be shown that the error of the nomogram remains unchanged during the process of transformation.

METHOD OF PROCEDURE WITH EXPERIMENTAL DATA.

Data obtained from observations are contaminated with errors of measurement which can never be removed entirely. So the observed values of a bi-variate relation cannot be represented by a single surface. If the equation of the relation under observation is not known, as is usually the case, adjustment by the method of least squares is also impossible. In such case, when a nomogram is to be constructed, the data should be adjusted by the following tentative method.

Let the observed values of r corresponding to different values of t and u be denoted according to the scheme shown in Table I. Plot the values of r in the first row against the values of t as shown in Fig. 8 with a vertical scale as large as practicable; draw a smooth curve passing through most of the points and leaving only a few of them on either side; and take the values of r from the curve. These values will be de-

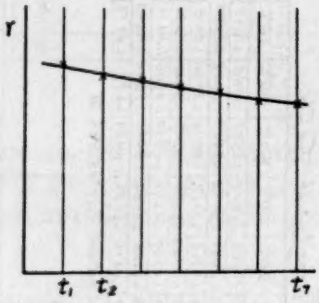


FIG. 8.

noted as a_1', a_2', . . . a_7'. The results of the method applied to the values in the other rows will be denoted in a similar manner.

The second course of treatment consists of plots with the adjusted values of r against the values of u; and from the curve determined by the values of a_2', b_2', c_2' . . ., g_2', readjusted values a_2'', b_2'', . . ., g_2'' may be obtained. Similarly determine the values a_6'', b_6'', . . ., g_6''.

Finally plot b_1, b_2'', b_3, b_4, b_5, b_6'' and b_7 against t again; draw smooth curve through the points of b_2'' and b_6'' and as many other points as possible; and read from the curve the corresponding values of r. These values will be denoted as b_1'', b_2'', . . ., b_7''. Similarly, determine the values of f_1'', f_2'', . . ., f_7''. The adjusted values marked with double apostrophes are to be used in the construction of the nomogram.

EXAMPLES.

The method of approximation proposed in this paper is unexpectedly simple. If its accuracy is sufficient for practical problems, it will greatly extend the scope of nomography. As it is impossible to formulate a general rule about the magnitude of the error of approximation,

examples will be worked out to indicate what it is capable of achieving in various actual cases.

Example 1. The construction of the nomogram for the relation between the density D of $CdCl_2$, the temperature t and its percentage of concentration per cent. as defined by the observed data given in Fig. 7.

The natural scale of D on which the values of the density are plotted with unit distance of 50 cm. is drawn first. Then the graduation points for $t = 0°$ C. and $t = 100°$ C. are assumed at 15 cm. from the D scale as shown, and are used to determine the graduation points of 25 and 45 per cent. by method of intersection. The coordinates of these two points with respect to the D scale as the Y-axis and the X-axis passing through the graduation point 1.400 on the D scale are found by measurements to be $X = -1.99$, $Y = -9.45$, $X = -2.93$ and $Y = 8.38$.

To avoid the enlargement of the error of plotting by the prolongation of short lines, two lines L_{25} and L_{45} are drawn parallel to the D scale and at distances 19.9 cm. and 14.65 cm. from the points of 25 and 45 per cent. respectively. The position of the point of 0° C. on L_{45} by projection from point 45 per cent. through the point at $D = 1.5893$ may be calculated as follows:

$$1.5893 - 1.4000 = 0.1893, \qquad 50 \times 0.1893 = 9.465,$$
$$9.465 - 8.38 = 1.085, \qquad 5 \times 1.085 = 5.425,$$

and

$$5.425 + 8.38 = 13.805.$$

The distance of the point 0° C. on L_{45} is therefore 13.805 cms. from the X-axis; and its position is accurately defined by this distance. Similarly, other points may be located on L_{45}; and the same set of points on L_{25} may be located in the same manner. The intersections of the corresponding lines give the graduation points of the t scale as shown.

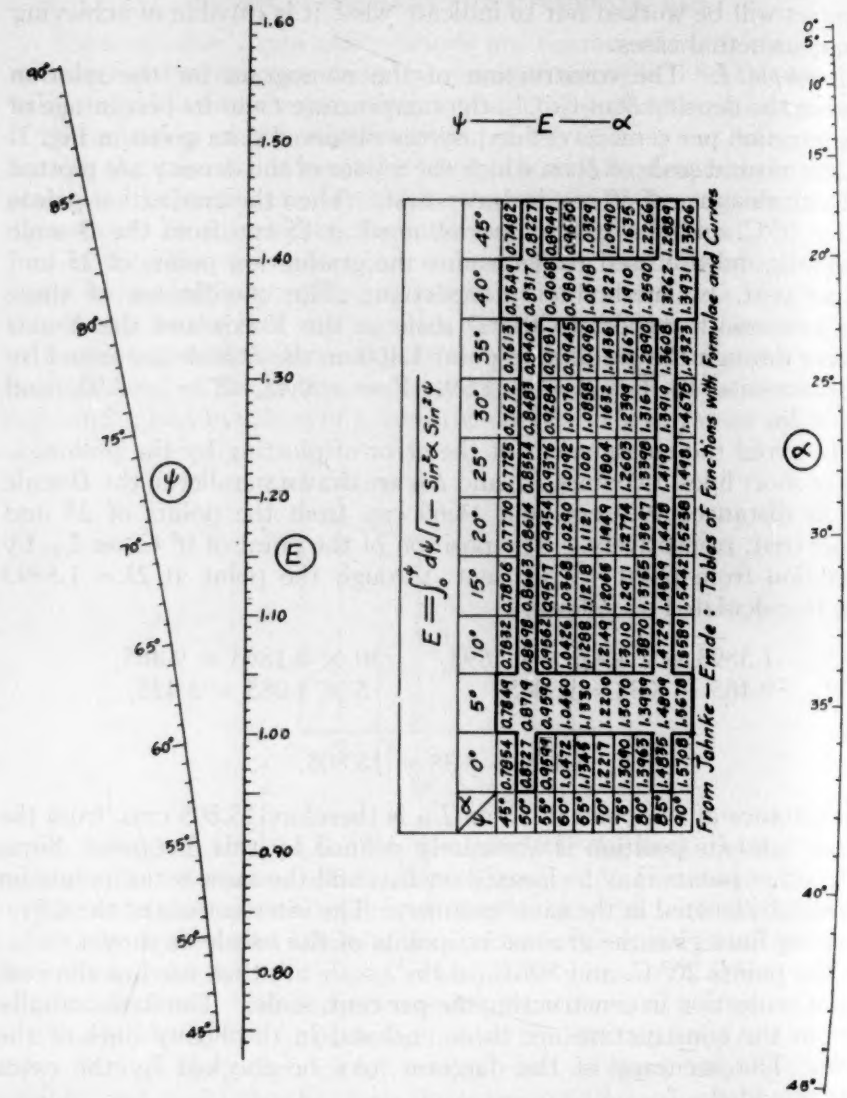
The points 20° C. and 80° C. of the t scale are then used as the centers of projection in constructing the per cent. scale. The data actually used in the construction are those enclosed in the heavy lines of the table. The accuracy of the diagram may be checked by the extra data provided.

Example 2. The construction of the alignment nomogram for the elliptic integral

$$E = \int_0^{\psi} \sqrt{1 - \sin^2 \alpha \sin^2 \psi}\, d\psi$$

from the data given in the table of Fig. 9.

The same method of construction has been used in this case; and the accuracy of the result has reached the level where the theoretical error is in the neighborhood of the error of construction.



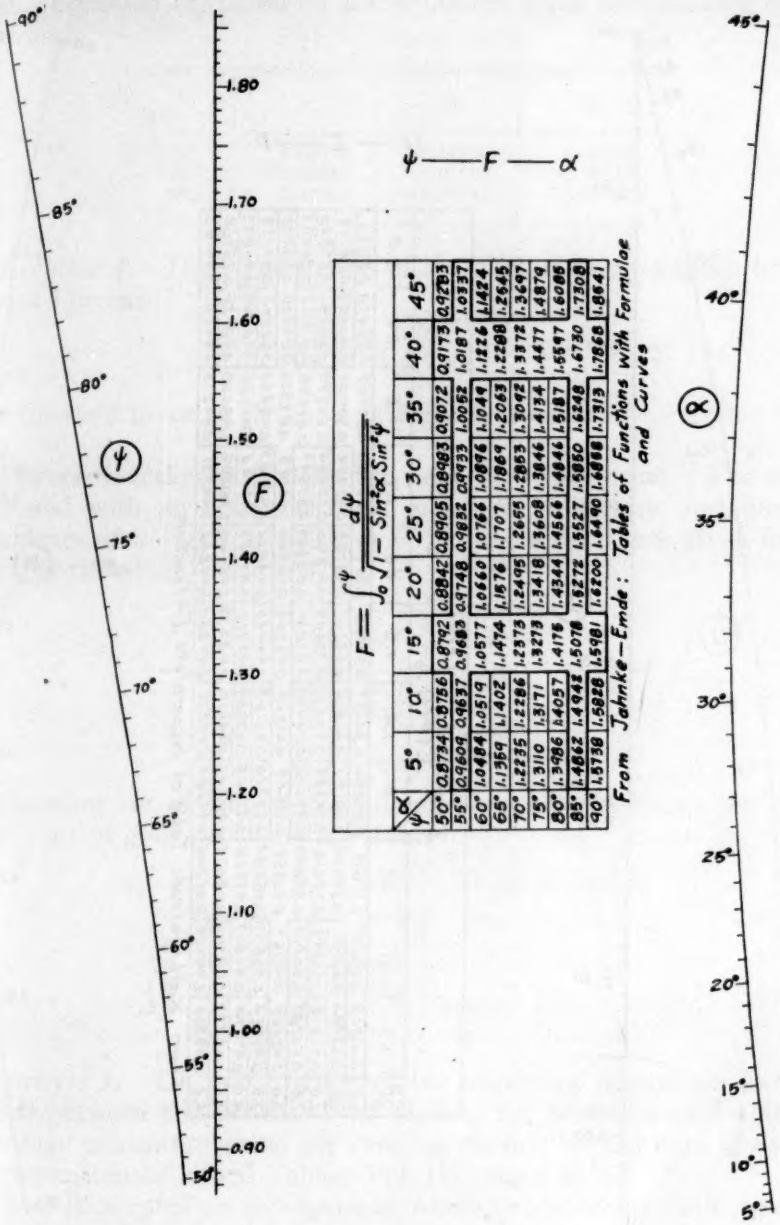
α \ ψ	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°
45°	0.7854	0.7849	0.7832	0.7806	0.7770	0.7725	0.7672	0.7613	0.7549	0.7482
50°	0.8727	0.8719	0.8698	0.8663	0.8614	0.8554	0.8483	0.8404	0.8317	0.8227
55°	0.9599	0.9590	0.9562	0.9517	0.9454	0.9376	0.9284	0.9181	0.9068	0.8949
60°	1.0472	1.0460	1.0426	1.0368	1.0290	1.0192	1.0076	0.9945	0.9801	0.9650
65°	1.1345	1.1330	1.1288	1.1218	1.1121	1.1001	1.0858	1.0696	1.0518	1.0329
70°	1.2217	1.2200	1.2149	1.2066	1.1949	1.1804	1.1632	1.1436	1.1221	1.0990
75°	1.3090	1.3070	1.3010	1.2911	1.2774	1.2603	1.2399	1.2167	1.1910	1.1635
80°	1.3963	1.3939	1.3870	1.3755	1.3597	1.3398	1.3161	1.2890	1.2590	1.2266
85°	1.4835	1.4804	1.4729	1.4599	1.4418	1.4190	1.3919	1.3608	1.3262	1.2889
90°	1.5708	1.5678	1.5589	1.5442	1.5238	1.4981	1.4675	1.4323	1.3931	1.3506

FIG. 9.

Example 3.—The construction of the alignment nomogram for the elliptic integral

$$F = \int_0^{\psi} \frac{d\psi}{\sqrt{1 - \sin^2 \alpha \sin^2 \psi}}$$

from the data given in the table of Fig. 10, with an investigation of its error.



$$F = \int_0^{\psi} \frac{d\psi}{\sqrt{1 - \sin^2\alpha \sin^2\psi}}$$

α \ ψ	5°	10°	15°	20°	25°	30°	35°	40°	45°
50°	0.8734	0.8736	0.8792	0.8842	0.8905	0.8983	0.9072	0.9173	0.9283
55°	0.9609	0.9637	0.9683	0.9748	0.9832	0.9933	1.0052	1.0187	1.0337
60°	1.0484	1.0519	1.0577	1.0660	1.0765	1.0896	1.1049	1.1226	1.1424
65°	1.1359	1.1402	1.1474	1.1676	1.1707	1.1869	1.2063	1.2288	1.2545
70°	1.2235	1.2286	1.2373	1.2495	1.2655	1.2853	1.3092	1.3372	1.3697
75°	1.3110	1.3171	1.3273	1.3418	1.3608	1.3846	1.4134	1.4477	1.4874
80°	1.3986	1.4057	1.4176	1.4344	1.4566	1.4846	1.5187	1.5597	1.6086
85°	1.4862	1.4948	1.5078	1.5272	1.5527	1.5850	1.6248	1.6730	1.7308
90°	1.5738	1.5828	1.5981	1.6200	1.6490	1.6868	1.7313	1.7868	1.8541

From Jahnke–Emde: Tables of Functions with Formulae and Curves

FIG. 10.

$$I = \frac{1}{\Gamma(p+1)} \int_0^{u\sqrt{p+1}} v^p e^{-v}\, dv$$

U \ p	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
1.0	0.41306	0.39430	0.37611	0.35850	0.34147	0.32502	0.30914	0.29383	0.27908	0.26490	0.25126
1.5	0.62584	0.61041	0.59605	0.58127	0.56659	0.55201	0.53755	0.52322	0.50902	0.49497	0.48109
2.0	0.77372	0.76399	0.75416	0.74423	0.73420	0.72409	0.71390	0.70364	0.69330	0.68291	0.67246
2.5	0.86782	0.86218	0.85643	0.85056	0.84458	0.83849	0.83229	0.82597	0.81955	0.81302	0.80639
3.0	0.92467	0.92167	0.91859	0.91543	0.91218	0.90885	0.90543	0.90193	0.89835	0.89468	0.89093
3.5	0.95784	0.95637	0.95484	0.95326	0.95163	0.94995	0.94822	0.94643	0.94459	0.94270	0.94075
4.0	0.97674	0.97607	0.97538	0.97465	0.97389	0.97310	0.97228	0.97144	0.97056	0.96965	0.96872
4.5	0.98732	0.98704	0.98675	0.98645	0.98613	0.98579	0.98540	0.98506	0.98468	0.98428	0.98386
5.0	0.99314	0.99305	0.99296	0.99284	0.99272	0.99260	0.99246	0.99231	0.99216	0.99200	0.99183

From Karl Pearson: Table of Incomplete Γ-Function

FIG. 11.

The construction of the diagram is clearly indicated by the data used. Equation (9) based on $K\alpha$ = infinity gives the following values of errors:

ψ \ α	5°	30°	45°
50°	0.00070	−0.00033	0.00076
70°	−0.00016	0.00069	−0.00062
90°	0.00013	−0.00041	0.00046

Example 4. The construction of the alignment nomogram for the gamma function

$$I = \frac{1}{\Gamma(p+1)} \int_0^{u\sqrt{p+1}} v^p e^{-v} dv$$

from the data given in the table of Fig. 11, with an investigation of its error.

The construction of the diagram needs no explanation. The errors calculated with an equation based upon K_u = infinity and lines of coincidence at $u = 1.0$, $u = 3.0$, $p = 1.2$ and $p = 2.0$ are given in the following table:

u \ p	1.0	1.6	1.7	1.8
2.0	−0.00090	0.00071		0.00040
4.0		−0.00011		
5.0			−0.00001	

The same set of data with the same lines of coincidence but with $K_u = 1.00797$ give the errors as follows:

u \ p	1.0	1.6	1.7	1.8
2.0	−0.00012	0.00010		0.00008
4.0		0.00004		
5.0			0.00003	

Example 5. The construction of the alignment nomogram for the relation between the density D of H_2SO_4, the temperature t and its percentage of concentration per cent. as defined by the data given in the International Critical Tables. Vol. III, pages 56–57.

Fig. 5 illustrates an arrangement whereby important data may be shown with greater accuracy. The diagram covering the relation from 1 to 50 per cent. is based on the lines of coincidence at 5 per cent., 45 per cent., 5° C. and 95° C. with $K_t = 0.9000$; and the other one covering the relation from 50 to 100 per cent. is based on the lines at 55 per cent., 90 per cent., 5° C. and 95° C. with K_t = infinity.

Example 6. Change of error of approximation due to change of scale.

The error of surface fitting given by the following data of the density of H_2SO_4

t \ %	5%	45%	70%	90%
5° C.	1.03545	1.35912	1.62453	1.83064
50° C.	1.01920	1.32506	1.58380	1.78290
95° C.	0.96789	1.29222	1.54582	1.73787

at (70 per cent., 50° C.) with lines of coincidence at 5 per cent., 90 per cent., 5° C. and 95° C. and a point of contact at (45 per cent., 50° C.) may be found to be 0.00094 which is equal to 1/918 of the total variation of the density (1.83064 − 0.96789).

The data may be transformed by any functional relation, and $\log_{10}(D + 5)$ gives

t \ %	5%	45%	70%	90%
5° C.	0.7807097	0.8033971	0.8211551	0.8344615
50° C.	0.7795388	0.8010646	0.8184766	0.8314154
95° C.	0.7758201	0.7988039	0.8159640	0.8285227

The error of the logarithm at (70 per cent, 50° C.) calculated under the same condition as before is 0.0000270 for which the corresponding error in D is 0.00041 which is equal to 1/2100 of the total variation of the density. In this case the accuracy is much improved by using the functional scale $\log_{10}(D + 5)$. Logarithmic transformation of the data sometimes reduces the error of fitting.

SIGNIFICANCE OF THE EXAMPLES.

The magnitude of the error of surface fitting cannot be predicted without going into actual calculations. The above examples serve the purpose of giving a general indication of the degree of accuracy and point to the following probabilities:

(a) A single alignment nomogram covering the ordinary ranges of variation of the variables of practical problems may be expected to be accurate to about 99.5 per cent.
(b) A suitable point of contact will generally reduce the error of approximation.
(c) A higher degree of accuracy may be obtained by dividing the region into several smaller areas to be represented by separate diagrams.
(d) Equation (8) may be used to compute many of the required values which could otherwise be obtained only by direct observation.

SYMMETRICAL BI-MODAL FREQUENCY CURVES.

BY

V. H. GOTTSCHALK, Ph.D.

Fifty years ago Karl Pearson, after futile attempts to split frequency curves into two component normal (Gauss) curves, declared the problem unsolvable and proceeded to develop the famous system of formulas now known as Pearson's Curves.

The splitting problem seems not to have been taken up since and may indeed be impossible, as experience with other theoretical solutions indicates, but it is surprising to find that some symmetrical distributions may be represented as the sum or difference of two normal components,

$$y = \frac{N_1}{\sqrt{2\pi\nu_{21}}} e^{-\frac{(x-s)^2}{2\nu_{21}}} + \frac{N_2}{\sqrt{2\pi\nu_{22}}} e^{-\frac{(x-t)^2}{2\nu_{22}}}$$

to give a fair approximation to the original data.

The above equation contains the six unknowns N_1, N_2, ν_{21}, ν_{22}, s and t, which Pearson determined by setting up equations obtained on integrating $\int_{-\infty}^{\infty} ydx$, $\int_{-\infty}^{\infty} yxdx$, up to $\int_{-\infty}^{\infty} yx^5dx$. This gives the six equations:

$$N_1 + N_2 = N, \tag{1}$$

$$N_1 s + N_2 t = N\nu_1, \tag{2}$$

$$N_1(\nu_{21} + s^2) + N_2(\nu_{22} + t^2) = N\nu_2, \tag{3}$$

$$N_1(3s\nu_{21} + s^3) + N_2(3t\nu_{22} + t^3) = N\nu_3, \tag{4}$$

$$N_1(3\nu_{21}^2 + 6s^2\nu_{21} + s^4) + N_2(3\nu_{22}^2 + 6t^2\nu_{22} + t^4) = N\nu_4, \tag{5}$$

$$N_1(15s\nu_{21}^2 + 10s^3\nu_{21} + s^5) + N_2(15t\nu_{22}^2 + 10t^3\nu_{22} + t^5) = N\nu_5. \tag{6}$$

From (1) and (2) we get:

$$N_1 = N\left(\frac{\nu_1 - t}{s - t}\right), \qquad \text{and} \qquad N_2 = N\left(\frac{s - \nu_1}{s - t}\right)$$

and these substituted in (3) and (4) give:

$$\nu_1(\nu_{21} - \nu_{22} + s^2 - t^2) - (t\nu_{21} - s\nu_{22} + s^2t - st^2) = \nu_2(s - t)$$

and

$$\nu_1(3s\nu_1 - 3t\nu_{22} + s^3 - t^3) - st(3\nu_{21} - 3\nu_{22} + s^2 - t^2) = \nu_3(s - t)$$

which may be simplified by choosing the origin at the mean ordinate,

thus making $\nu_1 = 0$, so that

$$t\nu_{21} - s\nu_{22} = -(\nu_2 + st)(s - t)$$

and

$$\nu_{21} - \nu_{22} = -\left\{\frac{\nu_3 + st(s+t)}{3st}\right\}(s - t)$$

from which a second pair of unknowns may be determined:

$$\nu_{21} = \nu_2 + st - \left\{\frac{\nu_3 + st(s+t)}{3t}\right\},$$

$$\nu_{22} = \nu_2 + st - \left\{\frac{\nu_3 + st(s+t)}{3s}\right\}.$$

When the above values of N_1, N_2, ν_{21}, and ν_{22}, together with $\nu_1 = 0$, are substituted in (5) and (6), the equations for the last pair of unknowns are found to be:

$$2s^2t^2(s+t)^2 + 4\nu_3 st(s+t) - \nu_3^2 - 3(\nu_4 - 3\nu_2^2)st - 6s^3t^3 = 0$$

and

$$2s^2t^2(s+t)^3 - (s+t)(5\nu_3^2 + 4s^3t^3) - 3(\nu_5 - 10\nu_2\nu_3)st + 20\nu_3 s^2t^2 = 0.$$

Eliminating $(s + t)$ between these two equations and using M and P to represent $(\nu_4 - 3\nu_2^2)$ and $(\nu_5 - 10\nu_2\nu_3)$, respectively, gives Pearson's nonic:

$$s^9t^9 + 3.5Ms^7t^7 + 1.5\nu_3^2 s^6t^6 + s^5t^5(3.75M^2 + 3P\nu_3) + s^4t^4(18.5M\nu_3^2 - 0.75P^2) + s^3t^3(12\nu_3^4 + 1.125M^3 - 4.5MP\nu_3) + s^2t^2(-2.625M^2\nu_3^2 - 3P\nu_3^3) - 4M\nu_3^4 st - \nu_3^6 = 0.$$

In reconsidering the problem, it seemed that a lower degree final equation might be obtained by finding a suitable substitute for the fifth moment, and a likely substitute appeared to be the first "half moment," which determines the position of the mean ordinate

$$\sum_{-\infty}^{0} yx = Ln_{1L} = \frac{N_1}{\sqrt{2\pi\nu_{21}}}\int_{-\infty}^{0} e^{\frac{-(x-s)^2}{2\nu_{21}}}\,xdx + \frac{N_2}{\sqrt{2\pi\nu_{22}}}\int_{-\infty}^{0} e^{\frac{-(x-t)^2}{2\nu_{22}}}\,xdx$$

$$= -\frac{N_1}{\sqrt{2\pi\nu_{21}}}\int_{0}^{\infty} e^{\frac{-(x-s)^2}{2\nu_{21}}}\,xdx - \frac{N_2}{\sqrt{2\pi\nu_{22}}}\int_{0}^{\infty} e^{\frac{-(x-t)^2}{2\nu_{22}}}\,xdx$$

$$= -\sum_{0}^{\infty} yx = -Rn_{1R}$$

but which does not appear in the above equations.

Carrying through this idea confirms Pearson's dictum that the problem is unsolvable, but it is precisely this integration over half the range that first showed the possibility of separating the data for a symmetrical distribution into two groups, each represented by a normal curve.

If we write:

$$L_S = \frac{N_1}{\sqrt{2\pi\nu_{21}}}\int_{-\infty}^{0} e^{\frac{-(x-s)^2}{2\nu_{21}}}\,dx, \qquad L_T = \frac{N_2}{\sqrt{2\pi\nu_{22}}}\int_{-\infty}^{0} e^{\frac{-(x-t)^2}{2\nu_{22}}}\,dx,$$

$$L_Sl_S = \frac{N_1}{\sqrt{2\pi\nu_{21}}}\int_{-\infty}^{0} e^{\frac{-(x-s)^2}{2\nu_{21}}}\,x dx, \qquad L_Tl_T = \frac{N_2}{\sqrt{2\pi\nu_{22}}}\int_{-\infty}^{0} e^{\frac{-(x-t)^2}{2\nu_{22}}}\,x dx$$

and L_Sl_{2S}, L_Tl_{3T}, etc., for higher moments, the following relations are obtained:

$$sL_S - L_Sl_S = \nu_{21}\left\{\frac{N_1}{\sqrt{2\pi\nu_{21}}}e^{-\frac{s^2}{2\nu_{21}}}\right\}, \tag{7}$$

$$tL_T - L_Tl_T = \nu_{22}\left\{\frac{N_2}{\sqrt{2\pi\nu_{22}}}e^{-\frac{t^2}{2\nu_{22}}}\right\}, \tag{8}$$

$$\begin{aligned} \nu_{21}L_S + sL_Sl_S &= L_Sl_{2S}, & \nu_{22}L_T + tL_Tl_T &= L_Tl_{2T},\\ 2\nu_{21}L_Sl_S + sL_Sl_{2S} &= L_Sl_{3S}, & 2\nu_{22}L_Tl_T + tL_Tl_{2T} &= L_Tl_{3T},\\ 3\nu_{21}L_Sl_{2S} + sL_Sl_{3S} &= L_Sl_{4S}, & 3\nu_{22}L_Tl_{2T} + tL_Tl_{3T} &= L_Tl_{4T}, \end{aligned}$$

and so on. The expressions in parentheses are the intercepts on the ordinate at the origin, for which the symbols y_{o1} and y_{o2} will be used.

L_S, L_T and L_Sl_S, L_Tl_T are not known, but their sums are obtained in the regular process of computing the full moments if Σy, Σyx, Σyx^2, etc., are summed separately to the left of the origin. That is, we have:

$$L_S + L_T = \sum_{-\infty}^{0} y = L, \tag{9}$$

$$L_Sl_S + L_Tl_T = \sum_{-\infty}^{0} yx = Ln_{1L}, \tag{10}$$

which with (7) and (8) give expressions for L_S, L_T, L_Sl_S, and L_Tl_T which when substituted in

$$L_Sl_{2S} + L_Tl_{2T} = \sum_{-\infty}^{0} yx^2 = Ln_{2L}$$

and

$$L_Sl_{3S} + L_Tl_{3T} = \sum_{-\infty}^{0} yx^3 = Ln_{3L}$$

give the two generally-valid equations:

$$\begin{aligned} \nu_{21}y_{o1}(\nu_{21} - \nu_{22} + st - t^2) + \nu_{22}y_{o2}(\nu_{21} - \nu_{22} + s^2 - st)\\ = L(n_{2L} - \nu_2)(s - t) - L(n_{1L} - \nu_1)(\nu_{21} - \nu_{22} + s^2 - t^2), \end{aligned}$$

and

$$\nu_{21}y_{o1}(s\nu_{21} + 2t\nu_{21} - 3t\nu_{22} + s^2t - t^3) + \nu_{22}y_{o2}(3s\nu_{21} - t\nu_{22} - 2s\nu_{22} + s^3 - st^2) = L(n_{3L} - \nu_3)(s - t) - L(n_{1L} - \nu_1)(3s\nu_{21} - 3t\nu_{22} + s^3 - t^3),$$

from which we get $y_{o1} + y_{o2} = Y_o$, yielding the following equation for the ordinate at the origin:

$$\begin{aligned} Y_o\nu_{21}\nu_{22}\{&2(\nu_{21} - \nu_{22})^2 + (t\nu_{21} - s\nu_{22})(s - t)\} \\ &= L(n_{3L} - \nu_3)\{(\nu_{21} - \nu_{22})^2 + (t\nu_{21} - s\nu_{22})(s - t)\} \\ &+ L(n_{2L} - \nu_2)\{-(\nu_{21} - \nu_{22})(s\nu_{21} - t\nu_{22}) \\ &- 2(\nu_{21} - \nu_{22})(t\nu_{21} - s\nu_{22}) - (t\nu_{21} - s\nu_{22})(s^2 - t^2)\} \\ &+ L(n_{1L} - \nu_1)\{-2(\nu_{21}^2 - \nu_{22}^2)(\nu_{21} - \nu_{22}) + (s - t)^2\nu_{21}\nu_{22} \\ &\qquad + 2(t\nu_{21} - s\nu_{22})^2 + st(t\nu_{21} - s\nu_{22})(s - t)\}. \end{aligned} \tag{11}$$

Although equation (11) and other "partial moment" equations do not graduate skew curves satisfactorily, the following considerations and computations show that acceptable results are obtained, simply and directly, when applied to symmetrical distributions.

A symmetrical bi-modal curve can arise in only two ways: (1) as the sum of two identical Gauss curves at equal distances on opposite sides of the mean ordinate, and (2) as two different Gauss curves added or subtracted with their maximum ordinates coinciding.

An astonishingly simple solution for the first case is obtained by putting $N_1 = N_2$, $\nu_{21} = \nu_{22}$, $s = -t$, in equations (1) to (4); since $\nu_1 = \nu_3 = 0$ for symmetrical curves, we get at once:

$$N_1 = \frac{N}{2}, \qquad \nu_{21} = \nu_2 - s^2, \qquad s^4 = \frac{3\nu_2^2 - \nu_4}{2}.$$

Another solution for s^2 may be obtained by substituting $\nu_{21} = \nu_{22}$, $s = -t$ in equation (11) and then eliminating ν_{21} by use of its equivalent, $\nu_2 - s^2$, as found above. The resulting expression is a quadratic in s^2, whose solution for the purposes of computation may be written:

$$s^2 = \frac{Ln_{1L} + Y_o\nu_2 \pm \sqrt{(Ln_{1L})^2 + Y_o(Ln_{3L} - \nu_2 Ln_{1L})}}{Y_o}.$$

The following three examples of symmetrical distributions used to test these two formulas and one later one are taken from the second edition, 1927, of Elderton's "Frequency Curves and Correlation."

The first case is illustrated by two examples: the first one, a synthetic double hump curve given on page 132, and the second, an actual distribution characterized by Elderton as "a rather interesting example of a symmetrical distribution" (page 85) "not capable of satisfactory graduation by the normal curve of error" (page 129).

For the double-hump curve, $N = N_1 + N_2 = 2031$; $\nu_2 = 4.610{,}537$; $\nu_4 = 46.016{,}741$, and hence $3\nu_2^2 - \nu_4 = 17.7544$, corresponding to $-s = t = 1.727$ and $\nu_{21} = \nu_{22} = 1.631$, with $N_1 = N_2 = 1015.5$, according to the simple formula. To use the "half moment" formula requires also $Y_o = 291$; $Ln_{1L} = -1{,}816$; $Ln_{3L} = -14{,}044$, and hence

$$s^2 = -1.630{,}010 \pm 4.410{,}848 = +2.780{,}838$$

or

$$s = -1.6676 \qquad \text{and} \qquad \nu_{21} = 1.8297.$$

In other words, the two formulas do not give identical results and it may be interesting to list both sets.

In Table I, the second column gives the y's computed by the equation

$$y = 794.3 \left\{ \frac{1}{\sqrt{2\pi}} e^{\frac{-(0.782x+1.351)^2}{2}} + \frac{1}{\sqrt{2\pi}} e^{\frac{-(0.782x-1.351)^2}{2}} \right\}$$

corresponding to the values of s and ν_{21} from the simple formula involving ν_4; the next three columns list y_1, y_2, and $y = y_1 + y_2$ computed from the equation

$$y = 750.7 \left\{ \frac{1}{\sqrt{2\pi}} e^{\frac{-(0.739x+1.233)^2}{2}} + \frac{1}{\sqrt{2\pi}} e^{\frac{-(0.739x-1.233)^2}{2}} \right\}$$

corresponding to the values given by the "half moment" formula. The last column lists y as given by Elderton.

TABLE I.

Double-Hump Curve (page 132).

x	y (calculated by 1st method)	(Calculated by 2nd method)			y (Elderton)
		y_1	y_2	$y = y_1 + y_2$	
0	254.4	140.0	140.0	280.0	291
1	302.1	42.8	265.1	307.9	303
2	314.2	7.6	290.6	298.2	286
3	193.4	0.8	184.4	185.2	193
4	65.4	0.04	67.8	67.8	78
5	13.0	0	14.4	14.4	10
6	1.2		1.8	1.8	—
7	—	—	0.13	0.1	—

This reproduces the double-hump nicely, especially by the "half moment" formula which fits the data the better of the two.

The second example of the first type of symmetrical curves is adapted for computation by the simple formula only, because the maximum ordinate is not given, and because the curve has $\nu_3 = 0.120{,}452$ instead of $\nu_3 = 0$ as is characteristic of symmetrical curves; it furthermore shows its skewness in that L is not equal to R, and Ln_{3L} is not equal to $-Rn_{3R}$. Since, however, further comparison of the two for-

mulas for the first case is desirable, the data for the "half moment" formula were obtained as follows: (*a*) Y_o was estimated from the graph of the curve (page 87) to be 463; (*b*) Ln_{3L} was replaced by $L(n_{3L} - \nu_3)$ as in equation (11) and thus $-3{,}258.177 - (852 \times 0.120{,}452) = -3{,}360.802$ which is practically identical with $R(\nu_3 - n_{3R}) = 831 \times 0.120{,}452) - 3{,}460.891$, as it should be, since $Ln_{3L} + Rn_{3R} = N\nu_3 = L\nu_3 + R\nu_3$.

The second column in Table II gives y as computed from the equation

$$y = 824\left\{\frac{1}{\sqrt{2\pi}}e^{\frac{-(0.9792x+0.9131)^2}{2}} + \frac{1}{\sqrt{2\pi}}e^{\frac{-(0.9792x-0.9131)^2}{2}}\right\}$$

obtained from the simple formula with

$$N_1 = N_2 = \frac{N}{2} = 841.5, \qquad s^4 = t^4 = \frac{3\nu_2^2 - \nu_4}{2} = \frac{1.51238}{2},$$

or $s = 0.9131$, and $\nu_{21} = \nu_2 - s^2 = 1.912{,}505 - 0.869{,}592 = 1.042{,}913$. The third, fourth, and fifth columns list y_1, y_2, and $y = y_1 + y_2$, respectively, as obtained from the equation

$$y = 734\left\{\frac{1}{\sqrt{2\pi}}e^{\frac{-(0.8722x+0.6746)^2}{2}} + \frac{1}{\sqrt{2\pi}}e^{\frac{-(0.8722x-0.6746)^2}{2}}\right\}$$

to correspond with the value of $s^2 = 0.597{,}993$ obtained from the quadratic formula with $Y_o = 463$; $Ln_{1L} = -Rn_{1R} = -963.734{,}780$; $L(n_{3L} - \nu_3) = R(\nu_3 - n_{3R}) = -3{,}360.802$, when L is taken as 852 and R as 831, with $\nu_3 = +0.120{,}452$.

The last column of Table II lists y as given by Elderton.

TABLE II.

"Symmetrical" Distribution (pages 85 and 129).

x	y (calculated by 1st method)	(Calculated by 2nd method)			y (Elderton)
		y_1	y_2	$y = y_1 + y_2$	
−5.498,515	0	0.06	0	0	—
−4.498,515	0.7	1.5	0	1.5	—
−3.498,515	14.2	17.4	0.3	17.7	11
−2.498,515	102.7	94.4	5.0	99.4	116
−1.498,515	301.3	239.7	41.1	280.8	274
−0.498,515	423.5	284.5	158.3	442.8	451
0.501,485	423.4	157.8	284.7	442.4	432
1.501,485	300.7	40.9	239.4	280.3	267
2.501,485	102.2	5.0	94.1	99.1	116
3.501,485	14.1	0.3	17.3	17.6	16
4.501,485	0.7	0	1.5	1.5	—
5.501,485	0	—	0.06	0	—

These results of the new formula compare favorably with the three graduations that Elderton considers excellent.

The second case for symmetrical curves requires $s = t = 0$, with, of course, $\nu_1 = \nu_3 = \nu_5 = 0$. These values substituted in equations (1) to (4) give only

$$N_1 = N\left(\frac{\nu_2 - \nu_{22}}{\nu_{21} - \nu_{22}}\right) = N\left(\frac{\nu_4 - 3\nu_{22}^2}{3\nu_{21}^2 - 3\nu_{22}^2}\right).$$

The first equation for determining ν_{21} and ν_{22} is, therefore,

$$3\nu_2(\nu_{21} + \nu_{22}) - 3\nu_{21}\nu_{22} = \nu_4$$

obtained by equating the two values of N_1, and the second equation is:

$$2Ln_{1L}(\nu_{21} + \nu_{22}) + 2Y_o\nu_{21}\nu_{22} = Ln_{3L}$$

obtained from equation (11) by the substitution of $s = t = 0$.

Solving for ν_{21} leads to the quadratic

$$\nu_{21}^2 - \left\{\frac{2Y_o\nu_4 + 3Ln_{3L}}{6(Y_o\nu_2 + Ln_{1L})}\right\}\nu_{21} + \left\{\frac{3\nu_2 Ln_{3L} - 2\nu_4 Ln_{1L}}{6(Y_o\nu_2 + Ln_{1L})}\right\} = 0,$$

whose two solutions represent ν_{21} and ν_{22}, which, together with the consequent values of N_1 and N_2, solves the problem.

Applying this to the example cited on page 89, by Elderton, as a "reminder that the 'normal curve' does not give entirely satisfactory results even with symmetrical distributions," gives the figures shown in Table III and computed by the equation

$$y = 924\left\{\frac{1}{\sqrt{2\pi}}e^{\frac{-(0.335x)^2}{2}}\right\} + 4{,}846\left\{\frac{1}{\sqrt{2\pi}}e^{\frac{-(0.669x)^2}{2}}\right\}$$

derived from the solution of the quadratic

$$\nu_{21}^2 - 11.124\nu_{21} + 20.186 = 0,$$

TABLE III.

Symmetrical Distribution (page 89).

x	y_1	y_2	$y = y_1 + y_2$	y (Elderton)
0	368.6	1,933.3	2,301.9	2,244
1	348.5	1,545.8	1,894.1	1,856
2	294.5	798.8	1,084.3	1,106
3	224.6	258.0	482.6	527
4	150.2	53.9	204.1	225
5	90.6	7.2	97.8	93
6	48.9	0.6	49.5	38
7	23.6	0.03	23.6	16
8	10.2	0	10.2	7
9	3.9	—	3.9	4
10	1.3		1.3	2
11	0.4		0.4	1
12	0.1		0.1	1

corresponding to the curve constants: $N = 10{,}000$; $\nu_2 = 4.0678$; $\nu_4 = 75.1846$; $Y_o = 2{,}244$; $Ln_{1L} = -7{,}491$; $Ln_{3L} = -76{,}055$. The two superimposed Gauss curves have $N_1 = 2{,}754.8$, $\nu_{21} = 8.8895$, and $N_2 = 7{,}245.2$, $\nu_{22} = 2.2345$, respectively.

In connection with this actual example of a second case symmetrical distribution, it may be stated that the algebraic sum of two Gauss curves is not another Gauss curve, but their product is:

$$\frac{N_1}{\sqrt{2\pi\nu_{21}}}e^{\frac{-(x-s)^2}{2\nu_{21}}} \times \frac{N_2}{\sqrt{2\pi\nu_{22}}}e^{\frac{-(x-t)^2}{2\nu_{22}}} = \left[\frac{N_1N_2}{2\pi\sqrt{\nu_{21}\nu_{22}}}e^{\frac{-(s+t)^2}{2(\nu_{21}+\nu_{22})}}\right]e^{\frac{-(x-b)^2}{2a}},$$

where

$$a = \left(\frac{t\nu_{21} + s\nu_{22}}{\nu_{21} + \nu_{22}}\right) \quad \text{and} \quad b = \left(\frac{\nu_{21}\nu_{22}}{\nu_{21} + \nu_{22}}\right).$$

In the absence of an example of a symmetrical distribution of the second kind in which two Gauss curves are subtracted, such a curve was synthesized and, to add interest, in such a way as to produce a double hump. The equation for this synthetic double-humped curve is:

$$y = \frac{20{,}000}{\sqrt{2\pi \times 4}}e^{-\frac{x^2}{2\times 4}} - \frac{8{,}000}{\sqrt{2\pi \times 1}}e^{-\frac{x^2}{2\times 1}}$$

for which $N = 12{,}000$; $\nu_2 = 5.9999$; $\nu_4 = 77.9821$ and $Y_o = 797.9$; $Ln_{1L} = -12{,}710.8$; $Ln_{3L} = -121{,}268.2$, giving $\nu_{21}^2 - 5.0336\nu_{21} + 4.2128 = 0$ by the second case quadratic and leading to

$$N_1 = 20{,}332; \qquad \nu_{21} = 3.9750;$$
$$N_2 = -8{,}332; \qquad \nu_{22} = 1.0587.$$

The difference between the synthetic curve and its graduation by the "half moment" formula for the second type of symmetrical bi-modal curves is shown in Table IV.

TABLE IV.

Synthetic Double-Humped Curve.

x	As graduated			As synthesized		
	y_1	y_2	$y = y_1 - y_2$	y_1	y_2	$y = y_1 - y_2$
0	4068.4	3230.6	837.8	3989.4	3191.5	797.6
1	3588.6	2014.0	1574.6	3520.7	1935.8	1584.9
2	2459.8	488.3	1971.5	2419.7	431.9	1987.8
3	1311.4	46.0	1265.4	1295.2	35.4	1259.8
4	543.2	1.7	541.5	539.9	1.0	538.1
5	175.2	0	175.2	175.3	0	175.3
6	44.0	—	44.0	44.3	—	44.3
7	8.6	—	8.6	8.7	—	8.7
8	1.3	—	1.3	1.3	—	1.3
9	0.2	—	0.2	0.2	—	0.2
10	0		0	0		0

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

MEASUREMENT OF TOTAL STRATOSPHERE OZONE.

It is recognized that the variation in the total amount of ozone in the stratosphere has a direct correlation with latitude and season, and that different types of air masses show variations in ozone content that are associated with their origin or movement. However, it is not known at present whether any direct relationship exists between such ozone variations and current weather conditions. The development of a suitable technique for routine measurement of total ozone is of great interest in order to establish possible correlations between ozone concentrations and weather phenomena. If such correlations do exist, ozone data may become very useful in weather forecasting.

At the request of the Naval Research Laboratory, Ralph Stair of the National Bureau of Standards recently made total-ozone measurements of the stratosphere over the Organ Mountains in New Mexico by means of ground-based equipment employing a photocell and selected filters. This method serves as a check upon ozone determinations by V-2 rocket flights from White Sands Proving Ground. The technique developed by the Bureau makes use of phototubes sensitive to ultraviolet radiation of wavelengths under 3400 A and filters with transmittances beginning at 2900 to 3100 A and increasing with wavelength. Since ozone is strongly absorbing within this region, the spectral energy distribution of sunlight reaching the earth's surface is greatly affected by its presence. Hence, when the transmittances of the filters are measured for sunlight (using the phototube as a detector), the observed value depends upon the solar energy distribution, and therefore is a function of the amount of ozone within the beam of sunlight under study.

The spectral energy curve of sunlight outside the earth's atmosphere may be calculated from measurements of filter transmittances of sunlight (for various altitudes of the sun) after correction has been made for Rayleigh scattering at the altitude and location of the observing station. Knowledge of the spectral response curve of the phototube, the spectral transmittances of the filters, and the spectral transmittances of selected amounts of ozone is required for this computation. When the spectral energy distribution of solar radiation outside the earth's atmosphere has been determined (within the spectral region extending from about 3000 to 3400 A) by this method, or has been obtained from data of other observers, the total amount of ozone in the stratosphere may be derived as a function of the observed filter transmittance of a

* Communicated by the Director.

single filter. After this relationship has been established, the total amount of ozone may be quickly determined from one filter transmittance measurement.

In the investigation at San Augustine Pass in the Organ Mountains, two phototubes and four filters were employed, providing eight independent determinations of the ozone value for the period of June 29 to July 4, 1947. The extremes of these eight determinations were about 0.19 and 0.24 cm. of ozone at normal temperature and pressure (that is, if all the ozone in the stratosphere were brought to sea level and reduced to 0 C., the total thickness would be 0.19 to 0.24 cm). During this period the mean value varied from about 0.20 to 0.22 cm. with changes in the type of air mass. A mean value was observed for continental tropical air, the higher value for a polar air mass, and finally the lower value for a tropical maritime air mass. The weighted mean of all determinations was 0.21 cm.

Additional measurements during the week of December 15, 1947 should give a winter value for total stratosphere ozone at the same location. These measurements will provide an independent set of ozone data for the time of the year when the sun is near its maximum declination south, while the mid-summer determinations furnish data corresponding approximately to the maximum northern declination of the sun. Positive conclusions of the ozone-weather relationship can be made only after more comprehensive data have been obtained and analyzed from many different observing stations, a project that is being planned by the Weather Bureau.

While extensive details are involved in the development of calibration curves and data for each phototube and set of filters, the Bureau's method becomes simple for routine operation at a particular observing station, after the original calibration data and curves have been prepared. Furthermore, since the auxiliary equipment consists only of a simple direct-current amplifier, the equipment for this type of work is inexpensive, and easily assembled and transported to suitable observing stations. The method is therefore recommended where routine measurements of ozone concentration are to be made at widely distributed stations.

NEW FLOW CALORIMETER MEASURES SPECIFIC HEATS OF GASES.

A new flow calorimeter for more accurate measurements of the specific heats of gases has been developed by R. B. Scott of the National Bureau of Standards. This apparatus was constructed in connection with a program sponsored jointly by the Bureau and the Office of Rubber Reserve for the determination of the thermodynamic properties of materials involved in the production of synthetic rubber and related substances. The greater precision of the new instrument is chiefly due to

the most complete elimination of heat leaks by various features of design. The small heat capacity of the calorimeter also results in improved accuracy through the easier and more rapid attainment of a constant temperature distribution within the system, since only under conditions of steady temperature is an accurate measurement possible.

Accurate data on specific heats and other thermal properties of gases are of value to chemical industry, where they are used in calculations that predict the results of chemical reactions. With sufficient knowledge of the thermodynamic properties of the compounds involved, it is possible to state the range of temperature within which a given reaction will occur and to estimate closely the relative amounts of the products.

Specific heat is most simply obtained experimentally by adding a measured amount of heat to a fixed mass and recording the temperature rise. However, the heat capacity per unit volume of a gas is so low that small accidental heat leaks are often comparable in size to the heat used in actually raising the temperature of the gas. Reduction of the ratio of heat loss or gain to heat input is thus one of the chief problems of gas calorimetry. In the calorimeter constructed at the National Bureau of Standards, this is accomplished by causing the gas to flow continuously at a uniform rate through an insulated chamber in which it is heated electrically. The heat capacity is then calculated from the heat input, the temperature rise of the gas, and the rate of flow. With gas calorimeters of this general type, a correction is usually made for heat leaks by making measurements at varying flow rates and extrapolating the apparent specific heats to infinite rate of flow, where such errors are negligible. In the new apparatus, however, special precautions have reduced heat leaks to such an extent that a correction for them is unnecessary. This advantage greatly lessens the number of necessary observations and also makes the results more trustworthy, since there are errors in calorimetric measurements that increase with increasing flow rate and cannot be removed by extrapolation to infinite flow.

The entire calorimeter is immersed in a constant-temperature bath. The gas whose specific heat is to be measured enters the system through a copper helix, in which it is brought to the temperature of the bath. It then flows into the calorimeter proper, where it traverses a labyrinth surrounding a constantan heater, and leaves the calorimeter through a throttle valve. The temperature rise within the calorimeter is determined by appropriately located thermocouple junctions.

A uniform rate of gas flow is obtained by maintaining a constant pressure on the inlet side of the calorimeter and a constant lower pressure at the outlet. Adjustment of the throttle valve in the outlet tube then provides the desired rate of flow, which is determined by weighing the gas collected in a condensing reservoir within a given time.

In the construction of the calorimeter, resistance to flow was reduced as much as possible, so that gases having low vapor pressures could be passed through the system at satisfactory rates. Since it was necessary to measure the specific heats of vapors that corrode copper and silver, all parts traversed by the gas were made of monel metal.

Heat leaks to or from the interior of the calorimeter are minimized in several ways. Conduction along the inlet and outlet tubes is reduced through the use of tubing of monel, a poor-conducting metal, having a wall thickness of 0.01 inch. Radiation along the tubes is prevented by a series of baffles, which in addition promote thermal equilibrium between the gas and the tube walls. Heat leaks due to either conduction or radiation are also lessened by providing a long section of tubing between the inlet and the heater, and by heating the outlet to the final temperature of the gas. Radiation to or from the inner portion of the calorimeter is prevented by a copper radiation shield kept at the temperature of the inner adjacent calorimeter wall by means of an electric heater. The necessary adjustments in the power to the heater are made in accordance with the readings of a three-junction thermocouple.

The accuracy of the apparatus was checked by measuring the specific heat of oxygen,[1] which had previously been accurately calculated at the Bureau from the latest spectroscopic data for this gas. The experimental values obtained at −30°, 40°, and 90° C. differed from the spectroscopic values by 0.06, 0.00, and 0.10 per cent. respectively. From a consideration of the data for oxygen, the probable error of the specific heats obtained with this calorimeter is estimated as 0.07 per cent.

The new calorimeter has been used at the Bureau by Ruth K. Cheney, Jane W. Mellors, R. B. Scott, and P. F. Wacker to determine the specific heats from −30° to 90° C. of isobutane, isobutylene, 1-butene, *cis*-2-butene, butadiene, styrene, and ethylbenzene.[1,2] Isobutane is a principal raw material for the production of aviation gasoline and is also converted to isobutene—a substance used in large quantities for the preparation of butyl rubber and isobutanol. 1-Butene, like the 2-butenes, is converted to butadiene, the major component of GR-S rubber. A knowledge of the thermodynamic properties of these materials is expected to aid in the more efficient operation of plants in which they are produced or used.

---

[1] Paul F. Wacker, Ruth K. Cheney, and Russel B. Scott, "Heat Capacities of Gaseous Oxygen, Isobutane, and 1-Butene from −30° C. to +90° C.," *J. Research*, NBS **38**, 651 (1947), RP 1804.

[2] Russel B. Scott and Jane W. Mellors, "Specific Heats of Gaseous 1,3-Butadiene, Isobutene, Styrene, and Ethylbenzene," *J. Research*, NBS **34**, 243 (1945), RP 1640.

THE FRANKLIN INSTITUTE.

STATED MONTHLY MEETING, WEDNESDAY, FEBRUARY 18, 1948.

The stated monthly meeting of The Franklin Institute was held on February 18, 1948 at 8:15 p. m. in the Lecture Hall. Mr. Richard T. Nalle, President, presided. There were approximately 200 persons in attendance.

The President announced that the minutes of the stated monthly meeting of December had been printed in full in the January issue of the JOURNAL and if no additions or corrections were offered from the floor the minutes would stand approved as printed. There was no dissent.

The Secretary was then called upon for his report. He announced the following members had been elected to Institute membership in the month of January:

Active	35
Associate	20
Student	43

It was announced that the total membership in The Franklin Institute as of January 31 was 5423.

The Secretary then informed the audience of two fascinating exhibits which are to be installed in the Institute in the early spring and summer. The one on Minting will be coincident with the coining of the new fifty cent piece which will bear the head of Benjamin Franklin on the obverse side and the Liberty Bell on the reverse side. Mrs. Nellie Taylor Ross, Director of the Mint in Washington, D. C., and former Governor of Wyoming, has been cooperating with us on this project.

The second exhibit is a model Railway which will be displayed about the middle of June. This exhibit is being sponsored by the Baltimore and Ohio Railway and is purported to be one of the finest models of its kind in the country.

Other exceptionally fine exhibits which will be featured in the Museum in 1948 include the Nuclear Energy Exhibit and the Bell Telephone Exhibit.

President Nalle then introduced Mr. L. F. Livingston, speaker of the evening. Mr. Livingston is manager of the Extension Division of the E. I. du Pont de Nemours & Company in Wilmington, Delaware. Mr. Livingston gave a very enjoyable talk which he appropriately called "Preview of Progress." He showed many types of products which can be made of Polythene and demonstrated its remarkable properties. Polythene, Mr. Livingston explained, is an inert substance affected by neither heat nor cold and can be readily colored. He told us of new applications made of Lucite, Cellophane, Nylon, Neoprene, together with a number of others and showed us samples of some which are now marketable.

Through Mr. Livingston's talk the audience gained a greater insight into aspects of the latest chemical research, though much is still in its experimental stage—as our speaker so adequately phrased it, "so far" these characteristics have been found.

At the conclusion of Mr. Livingston's excellent lecture the President adjourned the meeting with a rising vote of thanks to the speaker of the evening.

HENRY B. ALLEN,
Secretary

LIBRARY NOTES.

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. Orders received in the morning are filled the same day. The average cost for a print 9 × 14 inches is thirty-five cents.

The Library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from 9 A.M. until 5 P.M., Wednesdays and Thursdays from 2 P.M. until 10 P.M.

RECENT ADDITIONS.

AERONAUTICS.

LEY, WILLEY. Rockets and Space Travel. 1947.

ARCHITECTURE AND BUILDING.

HERKIMER, H., AND HAROLD HERKIMER. Air Conditioning. 1947.

BIBLIOGRAPHIES.

HOWE, JAMES LEWIS. Bibliographies of the Platinum Metals. 1947.

BIOGRAPHIES.

FEUCHTWANGER, LION. Proud Destiny. 1947.
HERRICK, CHEESMAN A. Stephen Girard, Founder. 1945.

CHEMISTRY AND CHEMICAL TECHNOLOGY.

BROWNLEE, KENNETH A. Industrial Experimentation. 1947.
INDIA RUBBER WORLD. Compounding Ingredients for Rubber. 1947.
KRUMBHAAR, WILLIAM. Coating and Ink Resins. 1947.
LITTLE, ROBERT W., ed. Flameproofing Textile Fabrics. 1947.
MERSEREAU, SAMUEL FOSTER. Materials of Industry. 1947.
RICHTER, VICTOR VON. Organic Chemistry. Third Edition. 1947.
SPRING, HARRY M., JR. Pressure Vessels for Industry. 1947.
WERTHEIM, EDGAR. A Laboratory Guide for Organic Chemistry. Second Edition. 1940.

CIVIL ENGINEERING.

LOCHNER, H. W., AND COMPANY. High Plan for Shreveport, Louisiana. 1947.

ELECTRICITY AND ELECTRICAL ENGINEERING.

CRARY, SELDEN B. Power System Stability, Volume 2. 1947.
LOEB, LEONARD B. Fundamentals of Electricity and Magnetism. Third Edition. 1947.
ROETGER, R. C. Industrial Electronics. 1947.

ENGINEERING.

KEATOR, FREDERIC W. Mechanical Laboratory Methods. Fifth Edition. 1947.

MATHEMATICS.

MCLACHLAN, NORMAN W. Theory and Applications of Mathieu Functions. 1947.
PETERSON, THURMAN S. Intermediate Algebra for College Students. 1946.
RIDER, PAUL R. College Algebra. 1943.

METALLURGY.

JEFFERSON, THEODORE B. Welding Encyclopedia. Twelfth Edition. 1947.

MILITARY ENGINEERING.

KARR, CHARLES L., AND C. K. KARR. Remington Handguns. 1947.

MILITARY SCIENCE.

STOUT, WESLEY W. Secret. 1947.

PETROLEUM.

WILSON, CHARLES MORROW. Oil Across the World. 1946.

PHOTOGRAPHY.

BUERGER, MARTIN JULIAN. The Photography of the Reciprocal Lattice. 1947.
SAYRE, IRENE H. Photography and Plate Making for Photo-Lithography. Third Edition. 1947.

PHYSICS.

CARSLAW, HORATIO S., AND J. C. JAEGER. Conduction of Heat in Solids. 1947.
CULVER, CHARLES AARON. Musical Acoustics. Second Edition. 1947.
FAIRES, VIRGIL MORING. Applied Thermodynamics. 1947.

RADIO.

KIVER, MILTON S. F-M Simplified. 1947.

RAILROADS.

BEEBE, LUCIUS. Mixed Trains Daily: a Book of Short-Line Railroads. 1947.

SCIENCE.

GAMOW, GEORGE. One, Two, Three . . . Infinity. 1947.

STEAM ENGINEERING.

STODOLA, A. Steam and Gas Turbines. Volumes 1 and 2. Sixth Edition. 1945.

PUBLICATIONS RECEIVED.

Fundamentals of Photographic Theory, by T. H. James and George C. Higgins. 286 pages, drawings and illustrations, 14 × 22 cm. New York, John Wiley & Sons, Inc., 1948. Price, $3.50.

Jet Propulsion Progress, by Leslie E. Neville and Nathaniel F. Silsbee. 232 pages, plates, drawings, illustrations and tables, 16 × 24 cm. New York, McGraw-Hill Book Company, Inc., 1948. Price, $3.50.

Magic Shadows, by Martin Quigley, Jr. 191 pages, drawings, illustrations and plates, 16 × 23 cm. Washington, D. C., Georgetown University Press, 1948. Price, $3.50.

Factual Communication, by L. O. Guthrie. 448 pages, illustrations, 15 × 22 cm. New York, Macmillan Co., 1948. Price, $3.50.

Theory of Servomechanisms, edited by Hubert M. James and others. 375 pages, drawings, 16 × 23 cm. New York, McGraw-Hill Book Co., 1947. Price, $5.00.

Textile Brand Names Dictionary. 1st Edition, 377 pages, 16 × 24 cm. New York, Textile Book Publishers, Inc., 1947. Price, $6.00.

Steel and Its Heat Treatment, by D. K. Bullens, Vols. 1 and 11, 489 pages and 293 pages, drawings, illustrations and tables, 15 × 23 cm. New York, John Wiley & Sons, Inc., 1948. Price, $10.00: Vol. 1—$6.00; Vol. 11—$4.00.

Timber Engineers' Handbook, edited by Howard J. Hansen. 882 pages, tables and drawings, 15 × 22 cm. New York, John Wiley & Sons, Inc., 1948. Price, $10.00.

Nomography, by A. S. Levens. 176 pages, drawings, 15 × 23 cm. New York, John Wiley & Sons, Inc., 1948. Price, $3.00.

Heat, by A. G. Worthing and David Halliday. 522 pages, tables, drawings and illustrations, 15 × 23 cm. New York, John Wiley & Sons, Inc., 1948. Price, $6.00.

Nicolaus Copernicus de Revolutionibus, translated by J. F. Dobson, assisted by S. Brodetsky. 32 pages, Port. and drawings, 16 × 25 cm. London, Royal Astronomical Society, 1947. Price, 3s. 6d. (Paper).

Thermodynamics, by Lester C. Lichty. 2nd Edition, 341 pages, plates, tables and drawings, 16 × 23 cm. New York, McGraw-Hill Book Co., 1948. Price, $4.50.

Thomas Jefferson Among the Arts, by Eleanor Davidson Berman. 305 pages, plates, 14 × 22 cm. New York, Philosophical Library, 1947. Price, $3.75.

A Survey of the Principles and Practice of Wave Guides, by L. G. H. Huxley. 328 pages, drawings, 14 × 22 cm. Cambridge, University Press; New York, The Macmillan Co.; 1947. Price, $4.75.

Computing Mechanisms and Linkages, by Antonin Svoboda, Ed. by Hubert M. James. 359 pages, tables, drawings and plate, 16 × 23 cm. New York, McGraw-Hill Book Co., 1948. Price, $4.50.

Crystal Rectifiers, by Henry C. Torrey and Charles A. Whitmer. 443 pages, tables, drawings and illustrations, 16 × 23 cm. New York, McGraw-Hill Book Co., 1948. Price, $6.00.

The American Annual of Photography of 1948, Volume 62, edited by F. R. Fraprie and Franklin I. Jordan. 216 pages, illustrations, 19 × 25 cm. Boston, American Photographical Publishing Co.; London, Chapman & Hall, Ltd.; 1947. Price, $2.00 (Paper).

BOOK REVIEWS.

ADULT EDUCATION FOR HOMEMAKING, by L. Belle Pollard. Second edition. 194 pages, 14 × 22 cms. New York, John Wiley & Sons, Inc., 1947. Price $2.75.

The material contained in this edition is a revision of an earlier book entitled "Adult Education in Homemaking."

It is designed for college use, as well as for the use of teachers who are responsible for the directing of the education of older youth and adults.

Written as a summary of experience it deals with the promotion, organization, supervision, and evaluation of educational programs.

HENRY M. MICHAEL.

ARCHITECTURAL CONSTRUCTION, by Theodore Crane. 414 pages, drawings and illustrations, 16 × 25 cms. New York, John Wiley & Sons, Inc., 1947. Price $6.00.

This book deals with the problem of making an appropriate choice for the structural portions of a building, as governed by the geographical location, site conditions, type of occupancy, equipment, and architectural design.

It is written for students in architecture, architectural engineering, and building construction, and portions of the text should prove of value to mechanical and electrical engineers who require a knowledge of modern building assemblies.

In the nine chapters of his book Mr. Crane furnishes the background for successful solution of constructional problems. He discusses building codes and design standards, the choice of framing, flooring and roofing materials, wall assemblies and foundations, as well as wide-span designs.

260 pertinent illustrations round out this comprehensive volume.

HENRY M. MICHAEL.

YARN AND CLOTH CALCULATIONS, by Lloyd H. Jackson. First edition. 196 pages, drawing, 13 × 22 cms. New York, Textile Book Publishers, Inc., 1947. Price $6.00 in U. S. and Canada, $7.00 in foreign countries.

In this short volume Mr. Jackson covers the essentials of yarn and cloth calculations, having in mind the needs of the textile student and the technician.

In its ten chapters he explains the yarn numbering systems and their conversions, ply yarns, grey cloth, warp, filling, and cloth calculations, the theory of fabric construction, calculations based on fabric analysis, and fabric analysis itself.

A list of problems has been added to enable the student to adopt the various formulae to practical applications.

HENRY N. MICHAEL.

FLIGHT ENGINEERING AND CRUISE CONTROL, by Harris G. Moe. 209 pages, drawings and illustrations, 15 × 23 cms. New York, John Wiley & Sons, Inc., 1947. Price $4.00.

The material contained in this volume is essentially written for those professional pilots and flight-crew members who, although technically and mechanically minded, do not have the necessary aeronautical engineering training to understand the subject in question, which hitherto had been presented in a highly technical manner. In his book, Mr. Moe treats this basically technical subject in such a manner that the reader need not be highly trained in order to understand it.

The contents of the book cover the following: altimeters and types of altitude, air speed, power and associated variables, power and airplane performance, supercharged engines, propellers, basic cruising techniques, flight planning and control, weight and balance, and jet propulsion.

The forward-looking approach of this book is evidenced by its discussion of air compressibility and jet propulsion.

HENRY N. MICHAEL.

PHOTOGRAPHIC FACTS AND FORMULAS, by E. J. Wall and Franklin I. Jordan. 364 pages, drawings, tables, 16 × 24 cms. Boston, American Photographic Publishing Co., 1947. Price $5.00.

The present edition of this book (the first edition was published some forty years ago) furnishes the photographic technician with a ready reference which contains the fundamental formulae and rules of photography. While not making any pretense of imparting purely scientific information, the descriptive matter is plain and concise.

This latest edition has been improved by the elimination of many pages covering obsolete processes, which have been replaced with material covering the most recent developments.

HENRY N. MICHAEL.

METEOROLOGICAL FACTORS IN RADIO-WAVE PROPAGATION. Report on a Conference held on 8 April 1946 at The Royal Institution, London, by The Physical Society and The Royal Meteorological Society. 325 pages, drawings, 17 × 26 cms. London, The Physical Society. Price 24 s. to non-members.

The papers contained in this report have been presented at the conference held in the Royal Institution, London, on April 8, 1946. They describe the various investigations concerned with the effect of the meteorological conditions of the lower atmosphere on the bending of very short radio waves transmitted through it.

The papers, twenty-one in all, deal with many aspects of the refractive index of the atmosphere and in this volume form a compact contribution to the study of radar.

HENRY N. MICHAEL.

IMPROVING SUPERVISION, by Frank Cushman and Robert W. Cushman. 232 pages, 13 × 19 cms. New York, John Wiley & Sons, Inc., 1947. Price $2.50.

Placing special emphasis on human relations problems, the authors have written this book for the purpose of making available to all supervisors ideas which they can apply to carry out their assignments more effectively. It has been especially designed for "follow-up reading" in connection with foreman and supervisor training programs.

The book should also be of interest to employers, managers, and training directors, since it deals with human engineering.

HENRY N. MICHAEL.

SAGA IN STEEL AND CONCRETE, NORWEGIAN ENGINEERS IN AMERICA, by Kenneth Bjork. 504 pages, illustrations, 15 × 23 cms. Northfield, Minn., Norwegian-American Historical Association, 1947. Price $4.00.

In a novel approach to the influence of national elements on the development of America, Dr. Bjork here treats of a particularly limited segment of the immigrants to this country—

Norwegian engineers. Although relatively small numerically in proportion to the whole immigration, this group is especially significant for two reasons: first, those immigrating constituted a very high percentage of the graduates of the technical schools in Norway, practically 25 per cent up to 1910, with some years running 50 per cent or more; and second, these engineers made many vital contributions to the technological building of the United States, a record which is presented in this volume in the lives and works of many of these engineers.

An introductory chapter surveys rapidly the beginnings of the industrial revolution in Norway, the establishment of technical schools there, and the factors which resulted in the emigration of so many of the graduates of those schools to the United States. The pioneers of this movement came in the 1860's, 1870's, and 1880's and found their way into various fields.

The chapter entitled "A Philadelphia Story" will undoubtedly prove most interesting to many JOURNAL readers for it concerns three men who settled in Philadelphia. The three are Tinius Olsen, who produced the first commercial testing machine; Henrik V. von Zernikow Loss, who succeeded in manufacturing an all-steel wheel and put this wheel on the all-steel railway car that had just been produced; and Mauritz C. Indahl, who was mainly responsible for developing the monotype machine into the useful versatile machine it is today. It is interesting to note that all three worked at some time for William Sellers, prominent Philadelphia manufacturer, as did so many others of these Norwegian engineers.

Then Dr. Bjork continues by devoting separate chapters to various types of engineering. In writing about bridges he gives primary attention to Thomas G. Pihlfeldt, who was instrumental in working out the Chicago type bascule bridge. In the field of tunnels the contributions seem even more significant for Olaf Hoff successfully applied the method of laying a tunnel in a prepared underwater trench, and Ole Singstad was responsible for developing a system of ventilation which has made possible the great automobile tunnels of the present.

Engineers who built skyscrapers, did metallurgical work, worked in the electric power field, built ships, and designed machines are among the others included. Of special interest is the chapter dealing with scientific management, which introduces Carl G. L. Barth, an assistant of Frederick W. Taylor, "the father of scientific management." Barth's vital contribution was the development of a slide rule suitable for ascertaining the correct cutting speed and feed for a lathe. After Taylor's death Barth became the leading exponent of the system with which he had been so closely associated during its period of formulation.

The activities of the Norwegian engineers in social and professional organizations among members of their own nationality are discussed. A concluding chapter offers a survey of their social philosophy. This volume is particularly important for the author's treatment of the contribution of one group of immigrants to the technological development of America. It is of interest also for the descriptions it contains of many outstanding engineering accomplishments and it will prove of considerable value as a source of biographical information about many engineers concerning whom such information may not be readily accessible.

G. E. PETTENGILL.

LECTURES DELIVERED AT PENNSYLVANIA SHORT COURSE FOR SEWAGE WORKS OPERATORS, 1945. 109 pages, illustrations, 17 × 25 cms. Pennsylvania Sewage Works Association, 1946.

LABORATORY GUIDE FOR SEWAGE WORKS OPERATORS, by D. Paul Rogers. 92 pages, illustrations, 17 × 25 cms. Pennsylvania Sewage Works Association, 1946.

These two volumes result from a course given in the spring of 1945 in Philadelphia sponsored by the United States Office of Education through the Engineering, Science and Management War Training Program. The content of the course was developed with the cooperation of the Pennsylvania Department of Health, the Pennsylvania Sewage Works Association and the Sewage Works Operators Association of Southeastern Pennsylvania.

The fifteen lectures were given by various individuals; some were state and city engineers, others were men connected with industrial concerns. The subject matter is designed for the sewage plant operator and considers sewage, reasons for its treatment, its characteristics, and the various treatment processes. Lectures were devoted to each of several types of equipment

including Imhoff tanks, intermittent sand filters, trickling filter, sludge digestion units, activated sludge plants, and chlorinators. Other lectures included the matter of sewage plant records, the effect of industrial waste on sewage treatment and the operation and maintenance of mechanical equipment.

The laboratory guide, while based on the original notes and outlines of the various instructors, has been rewritten by Mr. Rogers to obtain a uniform style of presentation. No attempt has been made to include tests not discussed in the course, nor does inclusion mean that every test is necessary at all sewage treatment plants. The method of treatment is clear and concise, comprising a definition of the test, indication of its purpose or use in sewage treatment works operation, specific step by step directions of procedure, and interpretation of the results by means of examples and notes.

As an introductory presentation of the fundamentals of sewage plant operation these volumes should be of value to the sewage works operator.

G. E. PETTENGILL.

TECHNICAL DRAWING PROBLEMS, by Frederick E. Giesecke, Alva Mitchell, and Henry Cecil Spencer. Second edition. 105 pages, drawings, 14 × 28 cms. New York, The Macmillan Co., 1947. Price $2.75 (Paper).

The 105 problems included in this volume are designed to cover the fundamental principles of technical drawing. They do not constitute a complete course in themselves, but are to be used in conjunction with a reference text. The "work plan" which prefaces the problems is designed to coordinate with the authors' text "Technical Drawing."

The individual problems are detachable.

HENRY N. MICHAEL.

ULTRAHIGH FREQUENCY TRANSMISSION AND RADIATION, by Nathan Marchand. 322 pages, drawings, 15 × 24 cms. New York, John Wiley & Sons, Inc., London, Chapman and Hall, Limited, 1947. Price $4.50.

The widespread use of ultrahigh frequencies presents many new problems in transmission and radiation which can be solved only by a thorough familiarization with the fundamentals.

Mr. Marchand has kept in mind the various applications and has written his book to meet the requirements of both the college student and the practicing engineer. The mks system is used throughout.

HENRY N. MICHAEL.

TECHNIQUES OF OBSERVING THE WEATHER, by B. C. Haynes. 272 pages, tables, drawings, and illustrations, 14 × 22 cms. New York, John Wiley & Sons, Inc., 1947. Price $4.00.

This book is written for high-school and college courses in elementary meteorology. It may be also used profitably by laymen who wish to take up the hobby of weather observing. For the guidance of the latter, Mr. Haynes includes a chapter on improvised weather stations. Otherwise, for the sake of uniformity needed in observing, the text follows closely the various United States Weather Bureau instructions on observations.

HENRY N. MICHAEL.

INDUSTRIAL HEALTH ENGINEERING, by Allen D. Brandt. 395 pages, drawings, tables, and illustrations, 15 × 24 cms. New York, John Wiley & Sons, Inc., 1947. Price $6.00.

Excessive concentrations of dusts, fumes, mists, gases, exposures to varying or unhealthy temperatures, to high humidity, harmful rays, noise, and to inadequate illumination, are of vital interest to the industrial health engineer. Dr. Brandt's book makes available to plant engineers, to construction and consulting engineering firms, and to others, in a condensed, practical and useful fashion, a summary of the information and data needed to determine what control measures should be employed and how control equipment should be designed.

Besides offering designs and control measures, Dr. Brandt classifies and evaluates the principal industrial atmospheric contamination.

The appendix contains tables of conversion, composition of some trade name solvents, size and characteristics of particular matters, and other pertinent data.

HENRY N. MICHAEL.

COLLEGE PHYSICS, by John A. Eldridge. Third edition. 720 pages, drawings, and illustrations, 14 × 22 cms. New York, John Wiley & Sons, Inc., London, Chapman & Hall, Limited, 1947. Price $4.50.

The third edition of Professor Eldridge's textbook retains the same approach as the earlier editions. In his own words, he "tried to write a text-book which will appeal to the student with only a general interest in the subject—non-science majors—and a text-book which will at the same time give a solid foundation for a student with a direct professional interest in physics." This resolution has been well carried out.

In this rewritten edition a somewhat greater number of worked-out examples are given and the order of presentation in mechanics is changed. In electricity considerable emphasis is given to electrostatics, fields, and potentials in space.

HENRY N. MICHAEL.

HEAT PUMPS, by Philip Sporn, E. R. Ambrose, and Theodore Baumeister. 188 pages, illustrations, 13 × 21 cms. New York, John Wiley & Sons, Inc., 1947. Price $3.75.

In this pioneer volume the authors have presented the fundamentals concerning the theory, construction, and operation of heat pumps. "The heat pump is the name commonly applied in present commercial practice to a year-round air-conditioning system employing refrigeration equipment in a manner which enables a surface to deliver usable heat to a space during the winter period and to abstract heat from the same space during the summer period." The idea of thus using the heat rejected from a refrigeration system is not new, but previously has found only a limited number of applications. Nor is it limited to heating as there are various potential industrial and manufacturing operations which employ the basic heat-pump principle.

The thermodynamic principles involved are outlined first in the book and then the authors proceed to a discussion of the four basic heat-pump systems, illustrating them by six different designs. The systems are air-to-air, water-to-air, air-to-liquid, and water-to-water, the first-named medium being the source of heat and the second being used to remove the heat from the condenser. Considerable attention is paid to design factors which must be considered, particularly climatic conditions and the source of heat. The use of government weather records in securing the climatological data is explained.

In the matter of equipment design the authors have limited their treatment to those aspects which are essential to the adequate development and choice of apparatus suitable for heat-pump service. This involves the consideration of compressors, heat transmission surfaces and fans. One chapter is devoted to an actual example detailing the working out of the equipment selection for a typical domestic installation of a heat pump.

The necessity for defrosting when air is used as a heat source is emphasized and several methods are outlined for accomplishing this end. Control systems are of various types, the all-electric being recommended as generally best for small systems, while the electric-pneumatic is best for the larger systems. The use of the heat pump for evaporation, distillation, and concentration in industrial applications is noticed and its importance in the last war in the distillation of drinking water from sea water is pointed out. Of especial interest are the comments on the economics of the heat pump and its potential effect on the power-system load curve. Illustrations and operating data of several installations are given in a concluding chapter. Bibliographical references are provided for further study.

This first book on the heat pump is of importance for its presentation of a system which may become of great significance in the heating and industrial development of America.

G. E. PETTENGILL.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

Manometric Determination of Phosphomonoesterase (Alkaline Phosphatase).—CHARLES A. ZITTLE. In the hydrolysis of monoesters of phosphoric acid a tertiary acid group is released. This group is not sufficiently acid ($pK_a = 12.4$) to release CO_2 from $NaHCO_3$ and accordingly does not provide the basis for a manometric procedure for following the hydrolysis of monoesters of phosphoric acid. These compounds, however, do have a property that makes it possible to follow their hydrolysis manometrically;—the secondary, as well as the primary, phosphoric acid group in the free acid is a weaker acid group ($pK_a = 6.8$) than is the same group in a great variety of mono-substituted derivatives of phosphoric acid (**1**,**2**,**3**,**4**). When hydrolysis of such compounds takes place the medium becomes more alkaline and in the presence of $NaHCO_3$ and an atmosphere of CO_2 some of the CO_2 is absorbed. The application of the method to the substrates β-glycerophosphate and guanylic acid is described herein. A limitation of the procedure is the narrow range of pH in the neighborhood of pH 6.8 to which it is applicable. This is considerably below the optimum for phosphomonoesterase (alkaline phosphatase) (pH 9 to 10), but it is in the neighborhood of the physiological range which is desirable for some types of studies (**5**).

EXPERIMENTAL.

In the hydrolysis of one equivalent of sodium β-glycerophosphate ($pK_2 = 6.34$) calculations show that about 0.21 equivalent of base would be released at pH 7.0, 0.24 equivalent at pH 6.8, 0.42 equivalent at pH 6.5. In the hydrolysis of one equivalent of guanylic acid ($pK_2 = 5.9$) about 0.39 equivalent of base would be released at pH 6.8. To be closer to the physiological range of pH and also to avoid the range where the base release changes considerably with pH, the experiments to be reported were performed at pH 6.8. As noted above, this is much below the optimum pH for the activity of this phosphatase.

The experiments were performed as follows: 0.5 cc. of 0.025 *M* $NaHCO_3$, 1.0 cc. of 0.43 *M* sodium β-glycerophosphate adjusted to pH 6.8 with *N* HCl, and water were placed in the bottom of Warburg flasks to make the total volume 3.5 cc. including the enzyme which was placed in the side arm. The enzyme, prepared from calf intestinal mucosa and partially purified (**6**), was used in a concentration of about 0.4 per cent in water in amounts of 0.2 to 0.8 cc. The atmosphere was 5 per cent CO_2—95 per cent N_2; the temperature was 37°. The results obtained are shown on the graph in Fig. 1. The absorption of

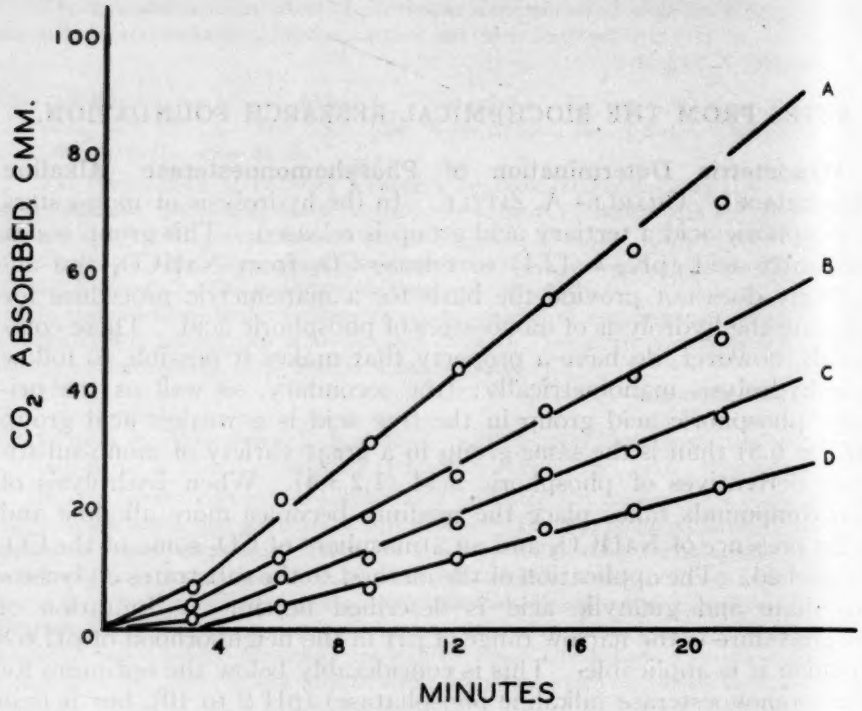


FIG. 1. Activity of phosphomonoesterase with glycerophosphate as substrate.

The rate of CO_2 absorption was measured manometrically with different amounts of the enzyme present. Curve A, 2.4 mg.; curve B, 1.6 mg.; curve C, 1.2 mg.; curve D, 0.8 mg.

CO_2 is proportional to the amount of enzyme used in each flask; the average is 166 cmm. (range 163 to 169) per 10 min. per 10 mg. of enzyme. The results with guanylic acid were similar but the rate of CO_2 absorption was about 20 per cent faster than with glycerophosphate.

Experiments were performed in which the amount of glycerophosphate was varied to determine the amount of substrate required for optimum activity of the enzyme; the amount used above (10 mg. per flask) provided a satisfactory excess.

The required pH was attained in other experiments with 100 per cent CO_2 and 0.6 cc. of 0.5 *M* $NaHCO_3$ with similar results.

Magnesium chloride in a final concentration of 0.004 *M* stimulated the enzyme about 10 per cent; larger concentrations were inhibitory.

Both sodium citrate and histidine in final concentrations of 0.02 *M* exerted more than 80 per cent inhibition in the system employed, which may be due to the ability of these compounds to bind positive ions necessary for activation of the enzyme. The actual amount of inhibition due to this effect is difficult to assess for both of these compounds have groups ionizing (pK of 5.4 and 6.0, respectively) in the neighborhood of the pH employed; hence, some of the base released by the hydrolysis

of glycerophosphate will be neutralized by buffer and consequently will not be available for the absorption of CO_2. An attempt was made to assess the magnitude of this effect by introducing small amounts of base, in the solid form, and measuring the CO_2 absorbed; Na_2CO_3, Na_3PO_4, and LiOH were employed. In an experiment with Na_3PO_4, 0.3 cc. of a 0.01 *M* solution was dried in the side arm of each of several flasks. When this amount of Na_3PO_4 was introduced into the system described previously with only $NaHCO_3$ and water to make the volume 3.5 cc., 65 cmm. of gas were absorbed which is a volume of about the expected magnitude. When, in addition to the $NaHCO_3$, the substrate glycerophosphate was present, only 19 cmm. of gas were absorbed. When with these substances was added 1.0 cc. of 0.1 *M* sodium phosphate, pH 6.8, or 1.0 cc. of 0.1 *M* sodium citrate, then the addition of Na_3PO_4 caused a *release* of gas (21 and 12 cmm., respectively). Comparable results were obtained with Na_2CO_3 and LiOH. This obviously does not represent the state of affairs when base is released by the hydrolysis of phosphate esters in the presence of buffering systems, and accordingly some other procedure must be found for assessing the effect of buffer compounds.

DISCUSSION.

The manometric method described for measuring phosphomonoesterase is convenient and useful where comparative assay data are desired. It has been used for following the fractionation of the phosphoesterase of calf intestinal mucosa which has both monoesterase and diesterase activity. The diesterase activity was also determined manometrically (**6**). Studies now in progress indicate that the two activities can be separated, that is, that diesterase and monoesterase are two distinct enzymes.

Preparations of phosphomonoesterase were assayed by both the present method and the method of Huggins and Talalay (**7**) which employs the phenolphthalein ester of phosphoric acid as substrate; an activity of 12,000 units per 1.0 g. of enzyme by the latter method corresponded to 100 cmm. per 10 min. per 10 mg. of enzyme by the manometric procedure.

SUMMARY.

A procedure for the manometric estimation of phosphomonoesterase is described.

REFERENCES.

(1) F. LIPMANN, *Advances in Enzymol.*, **1**, 99 (1941).
(2) W. D. KUMLER AND J. J. EILER, *J. Am. Chem. Soc.*, **65**, 2355 (1943).
(3) A. L. LEHNINGER, *J. Biol. Chem.*, **162**, 333 (1946).
(4) F. LIPMANN AND L. C. TUTTLE, *Arch. Biochem.*, **13**, 373 (1947).
(5) J. P. GREENSTEIN AND F. M. LEUTHARDT, *J. Natl. Cancer Inst.*, **7**, 1 (1946).
(6) C. A. ZITTLE, *J. Biol. Chem.*, **166**, 491 (1946).
(7) C. HUGGINS AND P. TALALAY, *J. Biol. Chem.*, **159**, 399 (1945).

CURRENT TOPICS.

X-Ray Speeds Chemical Comparisons.—Economical and rapid means of making chemical comparisons is provided by a new instrument known as the X-ray photometer. With this device it has been found possible to determine satisfactorily the tetraethyl lead content of gasoline, the concentration of an acid in water, the per cent. chlorination of a plastic, or the per cent. ash in coal. These determinations are made by measuring and comparing the X-ray absorption of a sample and a reference.

While the general principle of utilizing X-ray absorption as a means of making comparisons of chemical compounds has long been known, the particular method employed in the X-ray photometer is an outgrowth of wartime experience. When the Army Ordnance Department sought a rapid and reliable method of checking the explosive charge in hand grenade fuses to prevent premature detonation, engineers of General Electric then devised a method of X-ray testing which made it possible to check fuses accurately at a rate of 4,000 per hour.

Experience with this fuse-testing apparatus led to further experiments, and to the development of the X-ray photometer by the company's General Engineering and Consulting Laboratory. Essentially this device consists of a source of X-rays, a fluorescent screen, and multiplier phototube, an amplifier and an indicating instrument. The X-ray beam is interrupted by a synchronous motor-driven chopper in such a way that half of the beam passes alternately through each of two analyzer cells, one containing the reference and the other containing the sample. In the half of the beam passing through the reference there is placed also an aluminum attenuator disc, the angular position of which corresponds with a particular thickness of metal.

The X-rays from the two halves of the beam are received alternately on the fluorescent screen, from which the fluorescence is transmitted to the multiplier phototube. The output of this tube passes through an amplifier to a peak comparator where it registers on a microammeter as a d-c signal indicative of the difference in intensities of the two halves of the beam.

If the sample and the reference are identical the intensities of the two halves of the beam as received on the fluorescent screen are equal and the ammeter reading is zero without the use of the attenuator. If the sample and the reference are different it becomes necessary to introduce aluminum by means of the attenuator until the two halves of the beam become of equal intensity and the reading returns to zero. From the thickness of aluminum introduced into the reference half of the beam it is possible to determine empirically the proportion of certain elements in the sample as compared with the reference. In general the method is most applicable where there is a considerable difference in the atomic numbers of the main substance and the particular ingredient to be measured.

In actual operation the X-ray photometer has been found to work satisfactorily up to a speed of 6 minus samples per hour. Under most circumstances

the limiting factor has been found to be the time required for the preparation of the sample. Liquids, which must be measured as they are put into the analyzer cell, require more time than solids when the latter are specimens of uniform thickness. Powdered solids such as coal, which must be weighed into the analyzer cell, require more time than liquids. Preparation of the samples, however, can be arranged independently of the operation of the instrument. If this is done there is no difficulty in maintaining the 6 minus per hour rate regardless of the physical form of the material to be tested.

An outstanding advantage of this method of comparison is that it is independent of the physical state of the substance being tested, because the amount of X-ray absorption by a given mass of material is always the same, whether the material is hot or cold, gaseous, liquid, or solid. For example, the absorption per gram is the same in steam, in water and in ice.

Similarly, because X-ray absorption is an atomic property, measurements will be identical when an element is alone and when it is in chemical combination. An oxygen atom, for instance, will register the same whether it is as an element or in any oxygen compound.

Other advantages of the X-ray photometer method of chemical comparison are its speed and convenience. The complete apparatus is housed in a mobile cabinet of moderate size, and can be operated by any reasonably competent technician. Furthermore the analysis is made without loss or alteration of the sample tested.

In its present commercial form the X-ray photometers is available both as a manual type, but recording and self-balancing features can be added when desired.

R. H. O.

The Drain on Fertility.—"Our soil resources have been damaged to an alarming degree," Secretary Anderson warned recently. "And the drain of fertility has been accelerated by production for war and for relief of the postwar world food shortage. As soils lose fertility, farmers are more restricted in the crops they can raise. Also, the dangers of erosion increase. This in turn increases the danger of floods and of filling our streams, lakes and reservoirs with silt. Farming becomes a less profitable occupation and, as the mineral resources of the soil decrease, the hazards of poor quality national food supply increase. . . .

"Farmers want more fertilizer than is now available, despite the fact that they are using about three times as much plant food material as in 1920 and about twice as much as in 1940. The needs of the soil call for much more fertilizer than is now being used. The Department of Agriculture is working with the War Department to find ways of modifying existing facilities which may be used by private industry to meet additional demands for nitrogen fertilizer in this country."

R. H. O.

X-Ray Protection. (*Electrical Engineering*, Vol. 66, No. 10.)—An extensive program for determining the effectiveness of concrete as a protective barrier against million-volt wide-beam X-rays is being conducted by scientists at the National Bureau of Standards. Optimum wall thicknesses and most

desirable types of construction necessary for maximum short wave length X-ray protection are being investigated.

Broad beam X-rays which allow simultaneous examination of wide areas have come into extensive use for detecting flaws in all types of metal, and have created a special personnel protection problem. When a broad X-ray beam enters a thick concrete wall, it is scattered and rescattered many times with the result that a considerable fraction of the beam emerges on the other side, endangering personnel. Quantitative information on the amount of this scattering is part of the present investigation program.

To determine protection standards, a 1.5-million-volt X-ray machine has been converted so that it will produce beams of varying widths. It is mounted so that it can be directed downward into a radiation pit about six feet square and 20 feet below the target of the X-ray tube. Special instruments that can be shifted to various positions by remote control are used to explore the strength of the radiation in the pit. Both the control and the X-ray reading equipment are operated from a central control room surrounded by 18 inches of concrete and located 75 feet from the radiation pit.

Test slabs of concrete, each weighing 2½ tons and measuring six inches in thickness, are placed over the mouth of the pit one at a time, thus enabling investigators to vary the protective barrier from six inches to five feet in thickness. Since space and weight are of critical importance in many installations, arrangements also have been made for producing "sandwiches" of lead and concrete in alternate layers. At present, the best proportions between these two materials are entirely unknown.

Among the problems being investigated is the shielding of such machines as betatrons that operate at 50 million to 300 million volts. Because the beams emerging from such machines cover a very small angle (for the 50 million-volt betatron the angle is six or eight degrees), it may be possible to simplify the shielding problem.

For the direct beam, a very great thickness of concrete may be required. On the other hand, rays which are scattered sideways out of this beam are of a very much less penetrating character, and so it may be found that the protection problem for the side of the beam may not be seriously different from that at one or two million volts.

The collection and evaluation of data obtained in this investigation program can bring about sufficient protection at maximum economy. That the economic phase of high-voltage X-ray shielding is important is indicated by the fact that the cost of protection alone in a 50-million-volt medical or industrial installation may represent half to three quarters of the total cost of the equipment.

R. H. O.